Syllabus

Unit-1 Units & Dimensions

1.1 physical quantities – (Definition)

1.2 Definition of fundamental and Derived units, systems of units (FPS, CGS, MKS and SI units)

1.3 Definition of Dimension and Dimensional formulae of physical quantities.

1.4 Dimensional equations and Principle of homogeneity.

1.5 Checking the Dimensional correctness of physical relations.

Units & Dimensions

Science :-

The word Science is derived from a latin word "Scientia" which means "knowlegde" or "to know".

Hence Science is the systematic knowledge through the observation, experiment and verification.

Types of Science :-

There are two types of science

1. Physical Science
   (Physics, chemistry, maths etc)

2. Biological Science (Botany, zoology)

Physics :

Physics is the branch of science which deals with the study of nature and natural phenomena and the law governing it.

(or)

It is the law and principals that governs the universe.

Relation of physics with Mathematics:-

Mathematics is the language of physics so physics is incomplete without maths.

physical quantity:-

The quantities in terms of which the laws of physics can be expressed are called physical quantity.

Types of P.Q.

There are two types of P.Q.

1. Fundamental quantity
2. Derived quantity

Fundamental quantity:-

The quantitites in terms of which other quantities can be measured are called Fundamental quantity.

PQ	SI Unit	Symbol
1. Length	Meter	m
2. Mass	Kilogram	Kg
3. Time	Second	S
4. Temperature	Kelvin	K
5. Electric current	Ampere	A
6. Luminous intensity	candela	cd
7. Amount of Substance	mole	mol

Derived Quantity:

The quantities which are derived from fundamental quantities are called derived quantity.

Ex: Area, speed, Force, work, power volume etc.

Unit :-

The chosen standard of same kind taken for the measurement of physical quantity is called unit.

PQ = nu (n is numerical value and u is unit)

Ex: length of a wire = 15 meter

System of Units :-

These are 4 system of units

1. M.K.S System:

In this system fundamental quantities like length, mass and time are measured by meter (m), kilogram (kg.) and second (sec)

2. C.G.S. System:

In this system, length, mass and time are measured by centimeter (cm) gram (gm.) and second (sec)

3. F.P.S System :

In this system, length, mass and time are measured by foot (ft), Pound (P) and second (sec).

4. S.I. unit :

This is the international system of unit having 07 fundamental Unit and 02 Supplementary unit (plane angle - radian & Solid angle - steradian)

S.I. Prefixes

Microscopic

Fraction	Prefix	Symbol
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a
10^{-21}	zepto	z
10^{-24}	yocto	y

Macroscopic

Multiple	Prefix	Symbol
10	deca/deka	D/da
10^2	hecto/hecta	h/H
10^3	Kilo	k
10^6	mega	M
10^9	giga	G
10^{12}	teara	T
10^{15}	Peta	P
10^{18}	exa	E
10^{21}	zetta	Z
10^{24}	yotta	Y

Some other Units:

1. Astronomical unit (A.U.):

It is the mean distance of the earth from the sun.

$$1\ AU = 1.496 \times 10^{11}\ m$$

2. Light year (ly):

It is the distance travelled by light, in vacuum, in one year.

velocity of light in vacuum

$= 3 \times 10^8 ms^{-1}$

Time = 1 year = $365\frac{1}{4}$ day

$= 365\frac{1}{4} \times 24 \times 60 \times 60$ Seconds

1 light-year $= 3 \times 10^8 \times 365\frac{1}{4} \times 24 \times 60 \times 60$ m

$= 9.467 \times 10^{15}$ m (nearly).

3. Parallactic Second (ParSec):

It is the distance at which a length of one astronomical unit subtends an angle of one second of arc.

$$1 \text{ parsec} = 3.084 \times 10^{16} m.$$

1 micron $= 10^{-6}$ meter $= 10^{-4}$ centimeter

1 millimicron $= 10^{-9}$ meter $= 10^{-7}$ centimeter

1 Angstrom (A°) $= 10^{-10}$ meter $= 10^{-8}$ centimeter

1 fermi (1 fm) $= 10^{-15}$ meter $= 10^{-13}$ centimeter

π Radian $= 180^{\circ}$

$1^{\circ} = ?$ Radian

π Radian $= 180^{\circ}$

$180^{\circ} = \pi$ rad

$1^{\circ} = \frac{\pi}{180}$ rad

1 degree = 60 minutes

$$\left[\begin{array}{l} 60 \text{ min} = 1 \text{ degree} \\ 60' = 1^\circ \\ 1' = \left(\frac{1}{60}\right)^\circ \end{array}\right]$$

$1^\circ = 60'$

$1' = 60''$

$60'' = 1'$

$1'' = \left(\frac{1}{60}\right)'$

$1'' = \left(\frac{1}{3600}\right)^\circ$

1 horse power (hp) = 746 watt

1 litre = $1000 \text{ cm}^3 = \frac{1}{1000} \text{ m}^3$

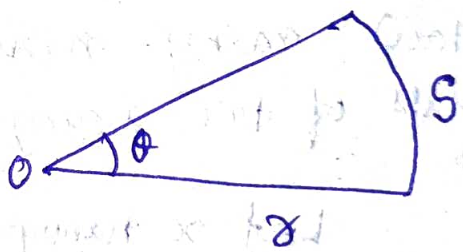
1 Tonne = 1000 kg , 1 Quintal = 100 kg

Supplimentary units :-

1. Radian :-

Radian describes the plane angle subtended by a circular arc as the length of the arc divided by the radius of the arc.

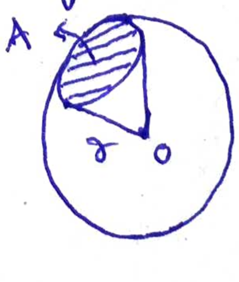
$$\boxed{\theta = \frac{s}{r} \text{ rad}}$$



2. Steradian :-

A Solid angle is the ratio between the area subtended and the square of its distance from the vertex.

$$\boxed{\Omega = \frac{A}{r^2} \text{ sr}}$$



Conversion among units :

1. Express a speed of 18 Km/hr in m/s.

Ans: Let, $18\ Km/hr = x\ \frac{m}{s}$

$$18\ \frac{Km}{hr} \times \frac{s}{m} = x$$

$$\Rightarrow 18 \cdot \frac{1000\,m}{3600\,s} \times \frac{s}{m} = 5\ \frac{m}{s}$$

Ans

2. Convert 10 m/s into km/hr.

Ans:

$$10\ m/s = x\ \frac{km}{hr}$$

$$x = 10\ \frac{m}{s} \times \frac{hr}{Km}$$

$$\Rightarrow x = 10\ \frac{m}{s} \times \frac{3600\ s}{1000\,m}$$

$$\Rightarrow x = 36$$

Ans = $36\ Km/hr$

3. How many nanograms are there in mass of two 2 exagram?

Ans:

Let x nanogram = 2 exagram

$$\Rightarrow x = 2\ \frac{exa}{nano}$$

$$\Rightarrow x = 2 \times \frac{10^{18}}{10^{-9}}$$

$$= 2 \times 10^{18} \times 10^{9}$$

$$= 2 \times 10^{27}\ \text{nanogram}$$

(9)

Dimensions :-

Dimensions of a physical quantity are the powers to which the fundamental units are to be raised in order to represent that physical quantity.

For dimension,

length is represented by [L]

Mass - [M]

Time - [T]

Temperature - [K]

Electric current - [A]

luminous intensity - [cd]

Amount of substance - [Mol]

E.g. Speed $= \frac{\text{distance}}{\text{time}} = \frac{[L]}{[T]} = [L^1 T^{-1}]$

$= [M^0 L^1 T^{-1}]$

So dimension of speed in Mass is 0, in length is 1, in time is -1.

Dimensional formula :-

It is an expression which indicates how and which of the fundamental quantities represents in the dimension of a physical quantity.

Generally

Dimensional formula represent

$[M^a L^b T^c]$

Where a, b, c are the dimensions of M, L and T respectively.

E.g.

1. Area = length × breath
$= L \times L = [L^2] = [M^0 L^2 T^0]$

2. Volume = lbh = $L \times L \times L = L^3 = [M^0 L^3 T^0]$

3. Density = $\frac{\text{Mass}}{\text{volume}} = \frac{M}{L^3} = [M^1 L^{-3} T^0]$

4. speed (or)
velocity = $\frac{\text{distance (displacement)}}{\text{time}}$

$= \frac{L}{T} = [M^0 L^1 T^{-1}]$

5. Acceleration = $\frac{\text{velocity}}{\text{time}} = \frac{L}{T^2} = [M^0 L^1 T^{-2}]$

6. Momentum = mass × velocity
$= M \times LT^{-1} = [M^1 L^1 T^{-1}]$

7. Force = mass × acceleration

$= M \times LT^{-2} = [M^1 L^1 T^{-2}]$

8. Work = Force × displacement
$= M^1 L^1 T^{-2} \times L^1 = [M^1 L^2 T^{-2}]$

9. Energy = amount of work = $[M^1 L^2 T^{-2}]$

10. Power = $\frac{\text{Work}}{\text{time}} = [M^1 L^2 T^{-2}] \times [T^{-1}]$

$= [M^1 L^2 T^{-3}]$

11. Pressure = $\frac{\text{force}}{\text{area}} = [M^1 L^1 T^{-2}] \times [L^{-2}]$

$= [M^1 L^{-1} T^{-2}]$

12. Angle = $\frac{\text{arc}}{\text{radius}} = [M^0 L^0 T^0]$

13. Angular velocity

$$= \frac{\text{angle}}{\text{time}} = [M^0 L^0 T^{-1}]$$

14. Time period = time $= [M^0 L^0 T^1]$

15. Frequency $= \frac{1}{\text{time period}} = [M^0 L^0 T^{-1}]$

Some Points about Dimensions :-

(i) The dimensions of a physical quantity do not depend upon system of units to represent that physical quantity.

(ii) Pure numbers and pure ratio donot have any dimensions.
e.g. refractive index, relative density, Strain etc.

(iii) Similar dimension can be added or subtracted but it does'not change the dimensions.

(iv) Logarithmic functions as $\log x$, e^x are the dimension less quantity :-

(v) Powers are dimension less.

(vi) The dimensions of two physical quantities may be same but the quantities need not be similar.

(vii) Dimension less quantity may have unit But unitless quantities are dimensionless. e.g. angle, Solid angle.

Dimensional equation :

It is an expression that indicates the relation of derived physical quantity with the fundamental quantities.

e.g. :

Speed $[V] = [M^0 L^1 T^{-1}]$

Force $[F] = [M^1 L^1 T^{-2}]$

Principle of homogeneity :

It states that dimensions of each of the terms of a dimensional equation on both sides should be same.

In a given equation,

$$A = B + C$$

According to this principle
dimension of A = dimension of B = dimension of C

e.g.-1
In an equation,

$$x = at + b,$$

If x in meter and t in second then find the dimension of a & b.

Ans:

$$[x] = [at] = [b]$$

$$\Rightarrow [x] = [at] \qquad \Rightarrow [x] = [b]$$

$$\Rightarrow [L^1] = [aT^1] \qquad \Rightarrow [b] = [M^0 L^1 T^0]$$

$$\Rightarrow [a] = \frac{[L^1]}{[T^1]}$$

$$\Rightarrow [a] = [M^0 L^1 T^{-1}]$$

So the dimension formula of $a = [M^0 L^1 T^{-1}]$
and $b = [M^0 L^1 b^0]$

Ex:2

Find the dimension and SI unit of $a, b \& c$ in $y = at^3 + bt^2 + c$, where y in meter and t in second.

Ans:

$$[y] = [at^3] = [bt^2] = [c]$$

$$[at^3] = [y] = [L^1]$$

$$[aT^3] = [L^1]$$

$$[a] = \frac{[L^1]}{[T^3]} = [M^0 L^1 T^{-3}] = ms^{-3}$$

$$[bt^2] = [y] = [L^1]$$

$$[b] = \frac{[L^1]}{[T^2]} = [M^0 L^1 T^{-2}] = m \cdot s^{-2}$$

$$[c] = [y] = [L^1]$$

$$[c] = [M^0 L^1 T^0] = m$$

(14)

Use of dimensional analysis:-

There are 03 uses

1. To check the correctness of a relation
2. To convert the value of physical quantity from one system to another
3. To derive a relation between various physical quantities.

1. To check the correctness of a relation

To check the correctness, dimension formula of each and every term on either side of an equation are calculated.

If the dimensional formula of each term are same then the relation is correct.

e.g.

Let us check the relation,

$$v = u + at$$

Ans: dimensional formula of 'v' = $[M^0 L^1 T^{-1}]$

dimensional formula of 'u' = $[M^0 L^1 T^{-1}]$

dimensional formula of 'at' =

$$[M^0 L^1 T^{-2}][T] = [M^0 L^1 T^{-1}]$$

This dimensional formula of all terms are same so the relation is correct.

(V-b) has the dimension of volume

∴ b = dimensions of volume

$b = [L^3]$

2. To convert the value of a physical quantity from one system of units to another.

It is based on the fact that.

Numerical value × unit = constant

So on changing units, numerical value will also gets changed.

If n_1 & n_2 are the numerical values of a given physical quantity and u_1 and u_2 be the units respectively in two different systems of units, then

$$n_1 u_1 = n_2 u_2$$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

Example: Convert Newton into dyne.

Soln: The Newton is the SI unit of force and has dimensions $[M^1 L^1 T^{-2}]$.

So $1N = 1 kg \cdot m / sec^2$

By using

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$= 1 \left[\frac{Kg}{gm}\right]^1 \left[\frac{m}{cm}\right]^1 \left[\frac{sec}{sec}\right]^{-2}$$

$$= 1 \left[\frac{10^3 gm}{gm}\right]^1 \left[\frac{10^2 cm}{cm}\right]^1 \left[\frac{sec}{sec}\right]^{-2}$$

$$n_2 = 1 \times 10^5 \text{ dyne}$$

Hence $1N = 10^5$ dyne

Example :-

Convert 5N in SI unit to dyne in CGS system.

Sol. Dimension formula of force $= [M^1 L^1 T^{-2}]$

Here $n_1 = 5N$

$n_2 = ?$

$$\Rightarrow n_2 = \frac{n_1 \times [M_1^a L_1^b T_1^c]}{[M_2^a L_2^b T_2^c]}$$

$$= 5 \times \frac{[1 Kg \times 1 m \times 1 s^{-2}]}{[1 gm \times 1 cm \times 1 s^{-2}]}$$

$$= 5 \times 1000 \times 100 = 5 \times 10^5 \text{ dyne}$$

$\therefore$ 5 Newton $= 5 \times 10^5$ dyne.

Example:

Convert 6 joule in SI unit to erg in CGS system.

Sol. Dimensional formula of work = $[M^1L^2$

Here $n_1 = 6$, $n = ?$

$$\Rightarrow n_2 = n_1 \times \frac{[M_1^a L_1^b T_1^c]}{[M_2^a L_2^b T_2^c]}$$

$$= 6 \times \frac{[1kg \times 1m^2 \times 1s^{-2}]}{[1gm \times 1cm^2 \times 1s^{-2}]}$$

$$= 6 \times 1000 \times 10000$$

$$= 6 \times 10^3 \times 10^4$$

$$= 6 \times 10^7 \text{ erg}$$

$\therefore$ 6 joule $= 6 \times 10^7$ erg

Example:

Convert a work of 1 joule into erg.

Solution:

3. To derive a relation between various physical quantities.

Using the principle of homogeneity of dimensions new relations among physical quantities can be derived if the dependent quantities are known and this dependence is of product type.

Example: Force depends upon mass and acceleration of a body then find the relation between them.

Ans:

Let $F \propto m^x$

$\propto a^y$

$\therefore F \propto m^x a^y$

$\therefore F = K m^x a^y$

By applying dimensional formula on both sides -

$$\Rightarrow [M^1 L^1 T^{-2}] = [M^1]^x \times [M^0 L^1 T^{-2}]^y$$

$$= [M]^x \times [M^0 L^y T^{-2y}]$$

$$\Rightarrow [M^1 L^1 T^{-2}] = [M^x L^y T^{-2y}]$$

$\Rightarrow x = 1, y = 1$

By putting the value of x and y, we get $F = kma$

Experimentally the value of $K = 1$

$\therefore F = ma$

Example:

Find the time period of a pendulum if it is a function of mass of the pendulum (m), length (l), acceleration due to gravity (g).

Sol.

Let $T = K m^x l^y g^z$

[K = dimension less const]

Now by substituting the dimensions of quantities -

$$[T^1] = [M^1]^x [L^1]^y [L T^{-2}]^z$$

or

$$[M^0 L^0 T^1] = [M^x L^{y+z} T^{-2z}]$$

Equating the exponents of similar quantities:

$x = 0$, $y + z = 0$ and $-2z = 1$

on solving we get $x = 0$, $y = \frac{1}{2}$ and $z = -\frac{1}{2}$

So the required physical relation becomes

$$T = K\sqrt{\frac{l}{g}}$$

The value of dimensionless constant K is found through experiments ~~so that~~

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Example:-

The position of a particle at time t is given by the relation

$$x(t) = \left(\frac{v_0}{\alpha}\right)\left(1 - e^{-\alpha t}\right),$$

where v_0 is a constant and $\alpha > 0$.

Find the dimensions of v_0 and α.

(Slove by using principle of homogeneity)

Sol.

Dimensions of $\alpha t = [M^0 L^0 T^0]$

$$\Rightarrow \alpha = \frac{[M^0 L^0 T^0]}{t}$$

$\therefore$ Dimension of $\alpha = [M^0 L^0 T^{-1}]$

Dimension of $\frac{v_0}{\alpha} = [L^1]$

$$\Rightarrow v_0 = [L^1] \times \alpha$$

$$\Rightarrow v_0 = [M^0 L^1 T^{-1}]$$

$\therefore$ Dimension of $v_0 = [M^0 L^1 T^{-1}]$

Gupta

Limitations of Dimensional Analysis:-

(1.) If dimensions are given physical quantity may not be unique. as many physical quantities have same dimensions.

e.g. If the dimensional formula of a physical quantity is $[M^1 L^2 T^{-2}]$, it may be work (or) energy (or) torque.

(2.) Numerical constants having no dimensions such as $1/2$, e^x, π etc. cannot be deduced by the methods of dimensions.

(3.) The method of dimensions cannot be used to derive relations other than product of power functions.

e.g. Relations $s = ut + \frac{1}{2}at^2$ (or) $y = a\sin\omega t$ cannot be derived by using this theory.

However the dimensional correctness of these can be checked.

(4.) The method of dimensions cannot be applied to derive formulas if a physical quantity depends on more than 3 physical quantities.

e.g.

Relation $T = 2\pi\sqrt{I/mgl}$ cannot be derived by theory of dimensions but its dimensional correctness can be checked.

(5.) Even if a physical quantity depends on 3 physical quantities, out of which two have same dimensions, the formula cannot be derived by theory of dimensions.

e.g. Formula for the frequency of a tuning fork $f = \left(\frac{d}{L^2}\right)v$ can not be derived by theory of dimensions but can be checked.

Syllabus

Unit-2 - Scalars and vectors

2.1 Scalar and Vector quantities,

Representation of a vector-examples, types of vectors.

2.2 Triangle and parallelogram law of vector Addition. Simple Numerical.

2.3. Resolution of vectors - Simple Numericals on Horizontal and vertical components.

2.4 vector multiplication. (Scalar product and vector product of vector).

Scalars And Vectors

physical quantities can be devided into two types. (Based on their directional properties)

1. Scalar Quantity :-

A physical quantity which can be described completely by its magnitude only and does not require a direction is known as a scalar quantity.

It obey the ordinary rules of algebra.

e.g. - Distance, mass, time, speed, temperature etc.

2. Vector Quantity :-

A physical quantity which requires magnitude and a particular direction, when it is expressed, is known as vector.

Vector quantity must obey the rules of vector algebra.

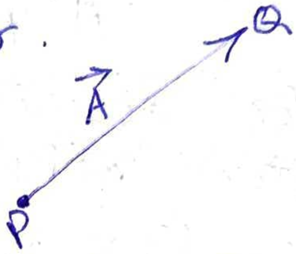
e.g. - For, displacement, velocity, acceleration etc.

Representation of a vector :

A vector is represented by a line headed with an arrow.

Its length is proportional to its magnitude.

Let $\vec{A}$ is a vector.



$$\vec{A} = \vec{PQ}$$

Magnitude of $\vec{A} = |\vec{A}|$ or A

The starting point of the arrowhead line (P) is called origin or point of application of the vector.

The end point of the arrowhead line is called tip or head of the vector.

The straight line itself shows the line of action of the vector.

And for direction we write its unit vector $\hat{A}$

So we can write $\vec{A} = |\vec{A}| \cdot \hat{A}$

$$\vec{A} = |\vec{A}| \cdot \hat{A}$$

$$|\vec{A}| = \frac{\vec{A}}{\hat{A}}$$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

(26) Types of vectors :-

(i) Polar vectors -

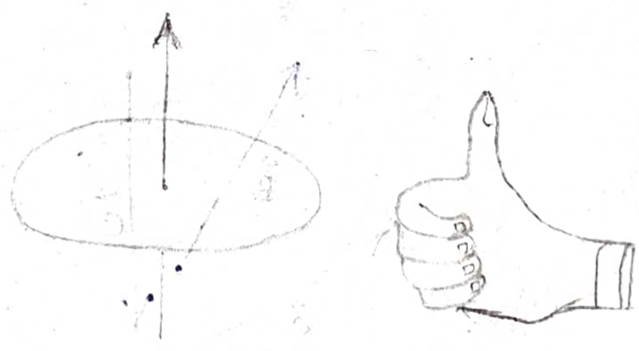
The vectors which have a starting point or a point of application are called polar vectors.

e.g. - Displacement, force etc.

(ii) Axial vectors -

The vectors which represents the rotational effect and act along the axis of rotation in accordance with right hand screw rule are called axial vectors.

e.g. - Angular velocity, torque, angular momentum etc.

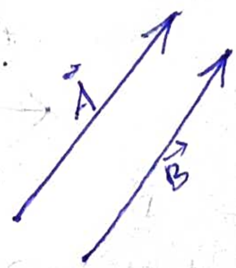


(iii) Equal vectors -

Two vectors are said to be equal if they have equal magnitude and same direction.

$$\vec{A} = \vec{B}$$

Angle betn two equal vector = 0°

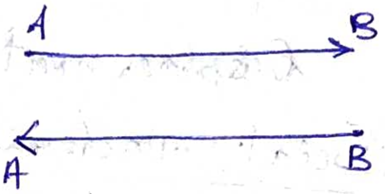


(iv) Opposite (or) Negative vectors-

The vectors which have equal magnitude but opposite direction are called opposite vectors.

$\vec{AB}$ and $\vec{BA}$ are opposite vectors

$$\boxed{\vec{AB} = -\vec{BA}}$$



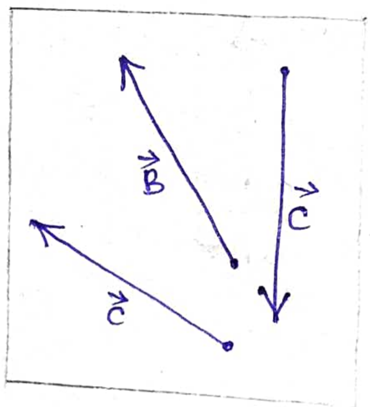
> Angle between two opposite vector = 180°

(v) Coplanar Vectors-

The vectors which are acting in the same plane are called coplanar vectors.

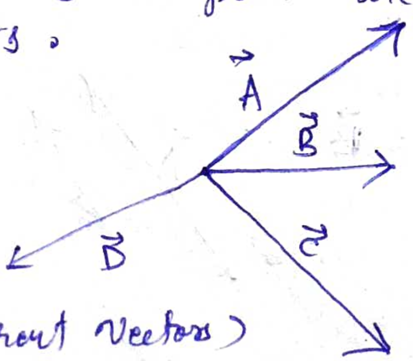
Note- Two vectors are always coplanar.

Here $\vec{A}$, $\vec{B}$ and $\vec{C}$ are coplanar vectors.

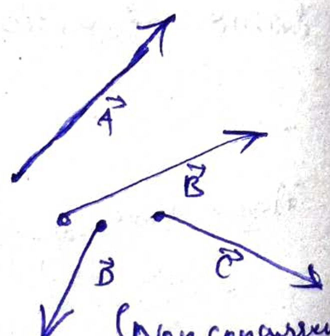


(vi) Concurrent Vectors-

The vectors which are acting on the same point are called concurrent vectors.



(Concurrent Vectors)



(Non concurrent vectors)

(vii) Null (or) zero vectors:-

A vector having zero magnitude is called null vector.

Example :-

Sum of two vectors is always a vector

So, $\vec{A} + (-\vec{A}) = \vec{0}$

Note - $\vec{A} + (-\vec{A}) \neq 0$

Here $\vec{0}$ is a zero or null vector.

Properties of a Null vector :-

(i) It has zero magnitude.

(ii) It has arbitary direction.

(iii) It is represented by a point.

(iv) When a null vector is added or subtracted from a given vector the resultant vector is same as the given vector.

(v) Dot product of a null vector with any vector is always zero.

(vi) cross product of a null vector with any vector is also a null vector.

(viii) Collinear vector:-

Vectors having a common line of action are called collinear vectors.

There are two types of collinear vectors

1. Parallel vectors

2. Anti Parallel vectors

1. Parallel vectors: $(\theta = 0°)$

If they ~~have~~ are acting in same direction.

2. Antiparallel vectors: $(\theta = 180°)$

If they are acting in opposite direction.

(ix) Localised vectors:-

Vector whose initial point is fixed is called localised vector (or) fixed vector.

e.g. - position vector.

(x) Non-localised vectors:-

Vector whose initial point is not fixed is called non-localised vector.

e.g. - force and momentum.

(xi) Unit vector:-

A vector having unit magnitude is called unit vector.

It is used by specify direction.

A unit vector is represented by $\hat{A}$.

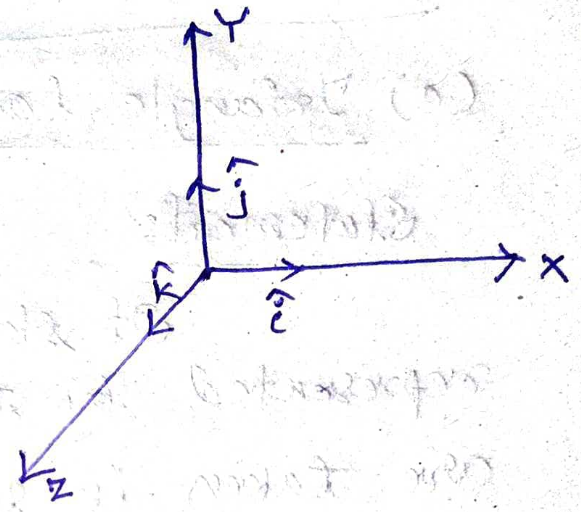
Unit vector in the direction of $\vec{A}$ is

$$\boxed{\hat{A} = \frac{\vec{A}}{|\vec{A}|}} \qquad \text{unit vector} = \frac{\text{vector}}{\text{Magnitude of vector}}$$

or, $\vec{A} = |\vec{A}|\hat{A} = A\hat{A}$

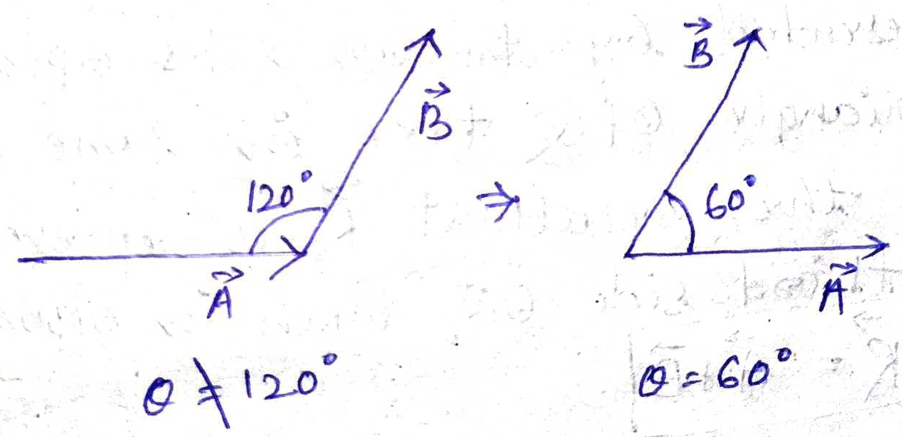
★ A unit vector is unit less and dimensionless vector and represents direction only.

★ In cartesian co-ordinates $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along x-axis, y-axis and z-axis.



Note :-

Angle between two vectors is that angle which is measured when they are joined from head to head (or) tail to tail.



Addition of two vectors:-

Vector addition can be performed by using following methods.

(i) Graphical method (ii) Analytical Method

(i) Graphical Method :-

(a) Triangle law of two vectors:-

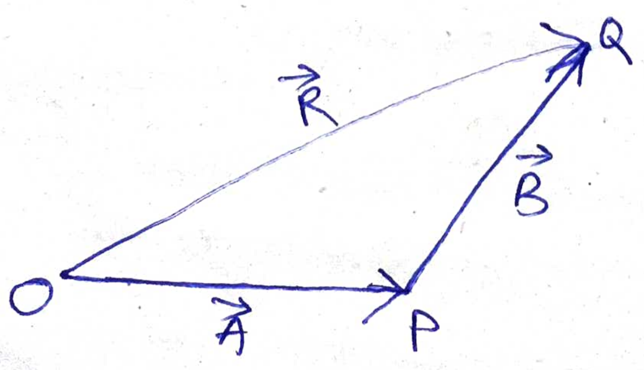
Statement:

It states that if two vectors are represented by the two sides of a triangle are taken in same order, then their resultant vector is represented by the third side of that triangle taken in opposite order.

Proof:

Consider two vectors $\vec{A}$ and $\vec{B}$ are represented by the two sides OP and PQ of triangle OPQ take in same order then the resultant $\vec{R}$ is represented by the third side OQ taken in opposite order.

$$\boxed{\vec{R} = \vec{A} + \vec{B}}$$



(ii) Analytically :-

Draw a perpendicular QT upon OP at point T.

Now in right angled triangle PTQ,

$\sin\theta = \dfrac{QT}{PQ}$

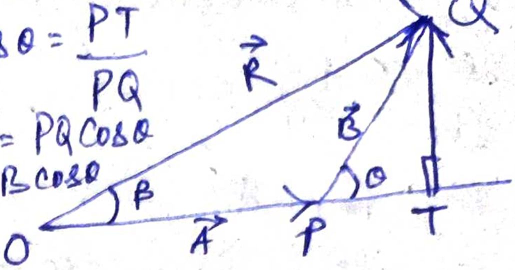
$\Rightarrow QT = PQ\sin\theta$

$= B\sin\theta$

$\cos\theta = \dfrac{PT}{PQ}$

$PT = PQ\cos\theta$

$= B\cos\theta$



Again in right angled triangle OTQ,

$$(OQ)^2 = (OT)^2 + (QT)^2$$

$$\Rightarrow (OQ)^2 = (OP+PT)^2 + (QT)^2$$

$$\Rightarrow R^2 = (A+B\cos\theta)^2 + (B\sin\theta)^2$$

$$\Rightarrow R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$

$$\Rightarrow R^2 = A^2 + B^2\cos^2\theta + B^2\sin^2\theta + 2AB\cos\theta$$

$$\Rightarrow R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$$

$$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\Rightarrow R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\Rightarrow \boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$$

This shows the magnitude of $\vec{R}$.

Again in right angled triangle OTQ,

$$\tan\beta = \frac{QT}{OT} = \frac{B\sin\theta}{A+B\cos\theta}$$

$$\Rightarrow \beta = \tan^{-1}\left(\frac{B\sin\theta}{A+B\cos\theta}\right)$$

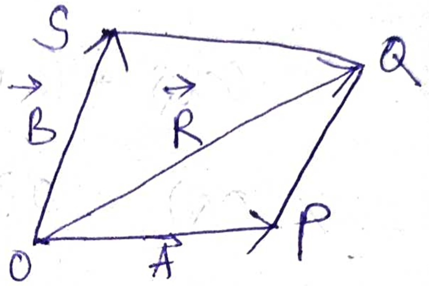
Here β shows the direction of $\vec{R}$.

(b) Parallelogram law of vector addition

Statement:

If two vectors are represented by two adjacent side of a parallelogram both pointing away from a common vertex then their resultant vector is represented by the diagonal of that parallelogram starting from that common vertex.

Graphically:-

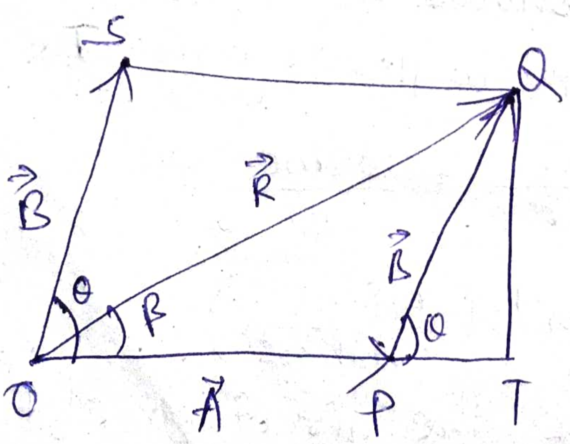


Consider two vectors $\vec{A}$ and $\vec{B}$ are represented by the two sides OP and OS of parallelogram OPQS then the resultant $\vec{R}$ is represented by the diagonal OQ away from the vertex O.

Here $\boxed{\vec{R} = \vec{A} + \vec{B}}$

Analytically:-

Draw a perpendicular QT upon OP at point T.



Proof:

Same as Triangle law of vector addition

Important points:-

(i) Resultant of two vectors will be maximum when they are parallel i.e. angle between them (θ) is zero.

$$R_{max} = \sqrt{A^2+B^2+2AB\cos 0^\circ}$$

$$= \sqrt{(A+B)^2}$$

or, $\boxed{R_{max} = A+B}$

(ii) Resultant of two vectors will be minimum when they are antiparallel i.e. angle between them (θ) is 180°.

$$R_{min} = \sqrt{A^2+B^2+2AB\cos 180^\circ} = \sqrt{(A-B)^2}$$

or, $\boxed{R_{min} = A-B}$

(Bigger - Smaller)

(iii) Vector addition is commutative.

$$\boxed{\vec{A}+\vec{B} = \vec{B}+\vec{A}}$$

(iv) Vector addition is associative.

$$\vec{A}+(\vec{B}+\vec{C}) = (\vec{A}+\vec{B})+\vec{C}$$

★ Vector addition is distributive $m(\vec{A}+\vec{B}) = m\vec{A}+m\vec{B}$

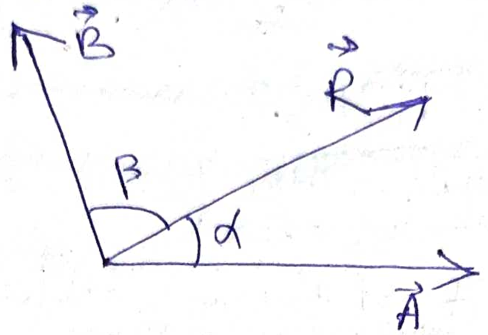
(v) Resultant of two vectors of equal magnitude will be at their bisector.

(vi) If vectors are of unequal magnitude then minimum three coplanar vectors are required for zero resultant.

(vii) If $|\vec{A}| > |\vec{B}|$ then $\beta > \alpha$.

Hence $\vec{R}$ will be more incline toward the vector of bigges magnitude.

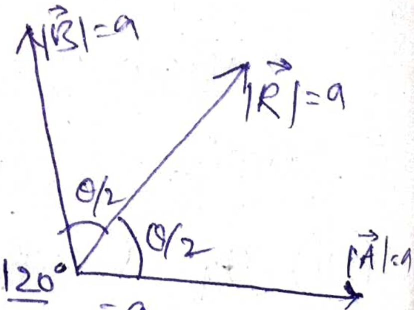
where $\vec{R} = \vec{A} + \vec{B}$



(viii) If vectors are of unequal magnitude and non-coplanar then, minimum 4 vectors are required for Zero resultant.

(ix) If two vectors have equal magnitude i.e. $|\vec{A}| = |\vec{B}| = a$ and angle between them is θ then resultant will be at the bisector of $\vec{A}$ and $\vec{B}$ and its magnitude is equal to $2a\cos\theta/2$

$|\vec{R}| = |\vec{A}| + |\vec{B}| = 2a\cos\theta/2$



Special case:-

If $\theta = 120°$

then $R = 2a\cos\theta/2 = 2a\cos\frac{120°}{2} = a$

i.e. $|\vec{R}| = |\vec{A}| = |\vec{B}| = a$

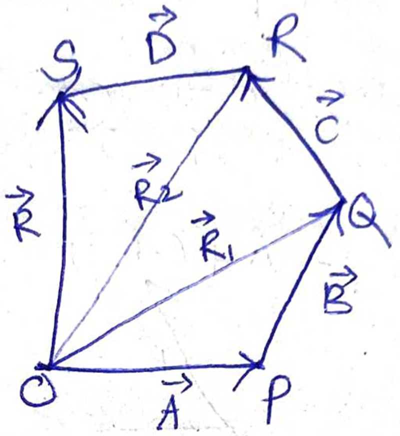
★ If resultant of two unit vector is another unit vector then the angle betn them $\theta = 120°$

Addition of more then two Vectors

Law of polygon :-

Statement :

If numbers of vectors are represented by the open sides of a polygon taken in same order then their resultant vector is represented by the closing side of that polygon taken in opposite order.



Here in polygon OPQRS , $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ are represented By the four side taken in same order, resultant $\vec{R}$ is represented by the closed side OS taken in opposite order.

$$\boxed{\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}}$$

Derivation :-

This law is based upon triangle law of vector addition.

In triangle OPQ , $\vec{R_1} = \vec{A} + \vec{B}$

On triangle OQR, $\vec{R}_2 = \vec{R}_1 + \vec{C} = \vec{A} + \vec{B} + \vec{C}$

On triangle ORS, $\vec{R} = \vec{R}_2 + \vec{D} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$

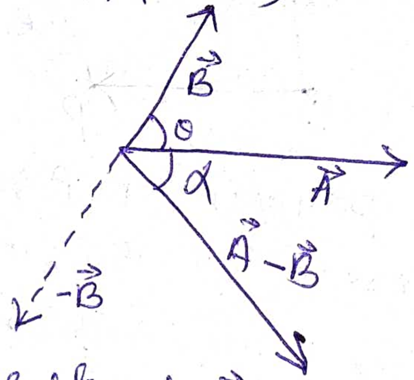
$\therefore \boxed{\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}}$

Subtraction of two vectors:

Subtraction of two vectors is same as addition of first vector with the negative of second vector.

Let $\vec{A}$ and $\vec{B}$ are two vectors. Their difference i.e. $\vec{A} - \vec{B}$ can be treated as sum of the vector $\vec{A}$ and vector $(-\vec{B})$.

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$



Hence to subtract $\vec{B}$ from $\vec{A}$, invert the direction of $\vec{B}$ and add to vector $\vec{A}$ according to law of triangle.

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

If α is the angle of difference vector from vector $\vec{A}$ than

$$\boxed{\tan\alpha = \frac{B\sin\theta}{A - B\cos\theta}}$$

Important points :-

1. The vector subtraction doesn't follow commutative law.

$$\boxed{\vec{A} - \vec{B} \neq \vec{B} - \vec{A}}$$

2. The vector subtraction doesnot follow associative law.

$$\boxed{(\vec{A} - \vec{B}) - \vec{C} \neq \vec{A} - (\vec{B} - \vec{C})}$$

3. If two vectors are equal in magnitude $|\vec{A}| = |\vec{B}| = a$ and θ is the angle between them, then

$$|\vec{A} - \vec{B}| = \sqrt{a^2 + a^2 - 2a^2\cos\theta} = 2a\sin\frac{\theta}{2}$$

Special case:

If $\theta = 60°$ then $2a\sin\theta/2 = a$

i.e. $|\vec{A} - \vec{B}| = |\vec{A}| = |\vec{B}| = a$

4. If two vectors are such that their sum and their difference vectors have equal magnitude then angle between the given vectors $(\theta) = 90°$.

i.e. $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ when $\theta = 90°$

5. If $\vec{A} + \vec{B} = \vec{A} - \vec{B}$ then $\vec{B} = \vec{0}$ (null vector)

6. If difference of two unit vectors is another unit vector then the angle between them is $60°$.

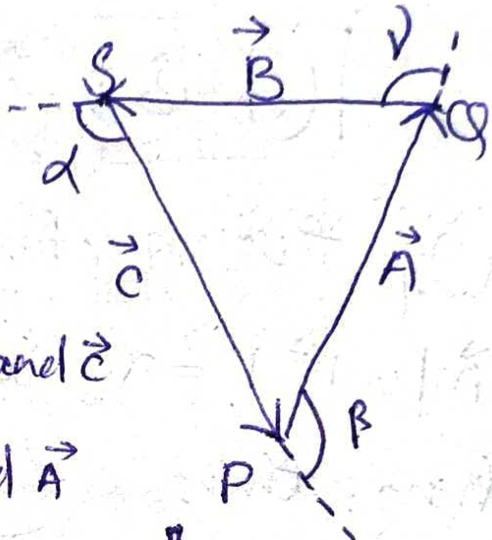
(39) Lami's Theorem -

If three vectors $\vec{A}, \vec{B}$ and $\vec{C}$ are represented both in magnitude and direction by the sides of a triangle taken in the same order, then

$\vec{A} + \vec{B} + \vec{C} = 0$.

and also -

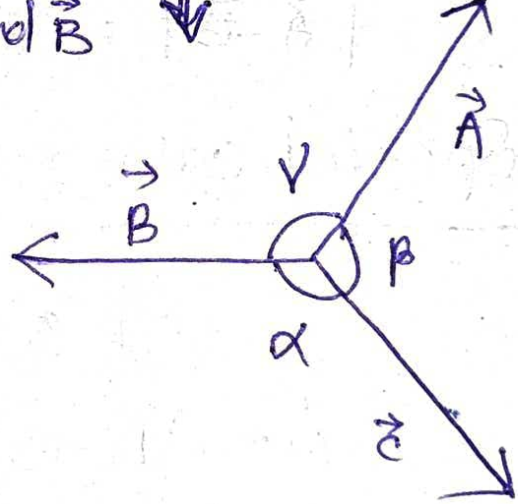
$$\boxed{\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}}$$



α = angle between $\vec{B}$ and $\vec{C}$

β = angle between $\vec{C}$ and $\vec{A}$

γ = angle between $\vec{A}$ and $\vec{B}$



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Resolution of vectors into rectangular Components

When a vector is splitted into components which are at right angle to each other than the components are called rectangular or orthogonal components of the vector.

In ΔOAB, $\frac{OB}{OA} = \cos\theta$

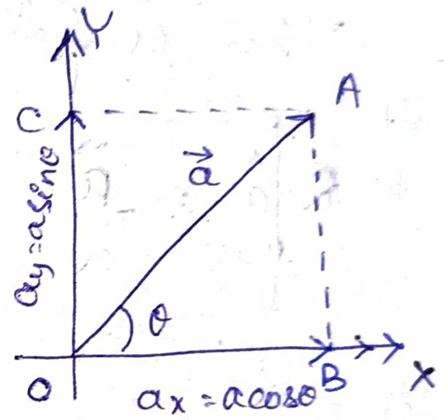
Or, $OB = OA\cos\theta$

Or, $\boxed{a_x = a\cos\theta}$

and $\frac{AB}{OA} = \sin\theta$

Or, $AB = OA\sin\theta = OC$

Or, $\boxed{a_y = a\sin\theta}$



Note:- If $\hat{i}$ and $\hat{j}$ denote unit vectors along OX and OY respectively then -

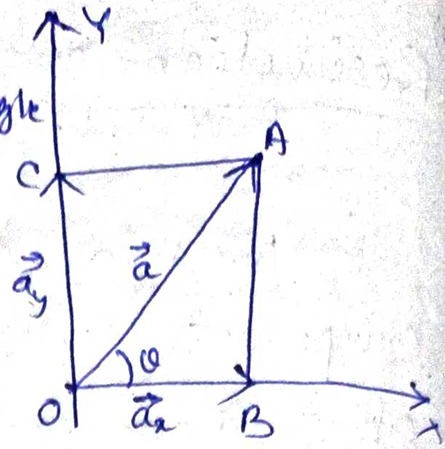
$\vec{OB} = a\cos\theta\,\hat{i}$ and $OC = a\sin\theta\,\hat{j}$

So that according to rule of vector addition $\vec{OA} = \vec{OB} + \vec{OC}$

Or, $\boxed{\vec{a} = a_x\hat{i} + a_y\hat{j}}$

$\Rightarrow \boxed{\vec{a} = a\cos\theta\,\hat{i} + a\sin\theta\,\hat{j}}$

In right angle triangle OAB



$$(OA)^2 = (OB)^2 + (AB)^2$$

$(\because AB = OC)$

$$(OA)^2 = (OB)^2 + (OC)^2$$

$$a^2 = a_x^2 + a_y^2$$

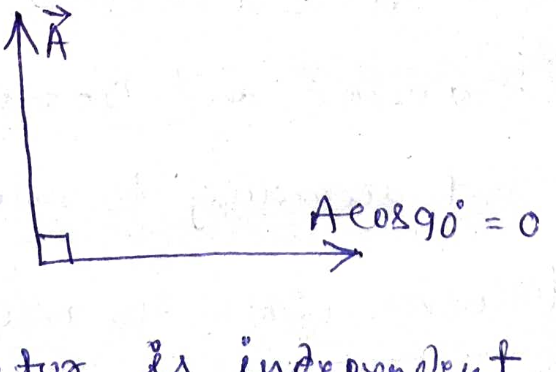
$$\boxed{a = \sqrt{a_x^2 + a_y^2}}$$

This is for magnitude

Unit Vector $\boxed{\hat{a} = \frac{\vec{A}}{a} = \frac{a_x\hat{i} + a_y\hat{j}}{\sqrt{a_x^2 + a_y^2}}}$

<u>Note:-</u>

1. The component of a vector along its perpendicular direction is always zero.



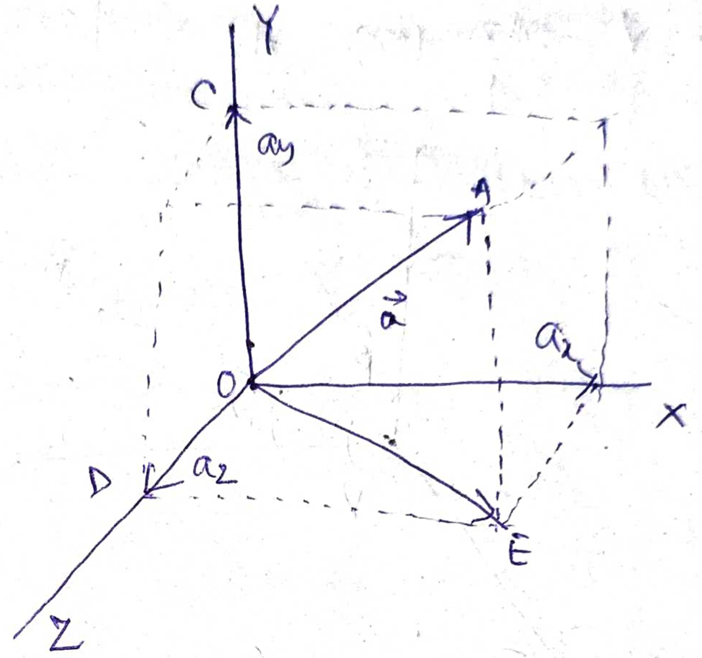
2. A vector is independent of the orientation of axes but the components of that vector depends upon the orentation of ~~axes~~ axes.

Rectangular components of a vector in three-dimensions :-

Now $\vec{OA} = \vec{a}$, $\vec{OB} = a_x\hat{i}$, $\vec{OC} = a_y\hat{j}$

and $\vec{OD} = a_z\hat{k}$

$\therefore \boxed{\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}$



Also $(OA)^2 = (OE)^2 + (EA)^2$

But $(OE)^2 = (OB)^2 + (OD)^2$

and $EA = OC$

$\therefore (OA)^2 = (OB)^2 + (OD)^2 + (OC)^2$

or, $a^2 = a_x^2 + a_y^2 + a_z^2$

Hence $\boxed{a = \sqrt{a_x^2 + a_y^2 + a_z^2}}$

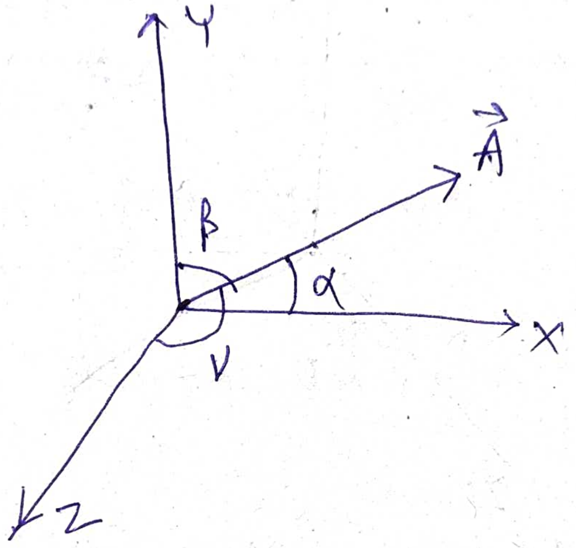
Directional cosines :-

Let $\vec{A}$ makes angle α with x-axis β with y axis and γ with z axis, then,

$$\cos\alpha = \frac{A_x}{A} \quad (\text{or}) \quad A_x = A\cos\alpha$$

$$\cos\beta = \frac{A_y}{A} \quad (\text{or}) \quad A_y = A\cos\beta$$

$$\cos\gamma = \frac{A_z}{A} \quad (\text{or}) \quad A_z = A\cos\gamma$$



Here $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are directional cosines of the vector.

By putting the values of A_x, A_y and A_z in equation $a^2 = a_x^2 + a_y^2 + a_z^2$

we get,

$$A^2 = A^2\cos^2\alpha + A^2\cos^2\beta + A^2\cos^2\gamma$$

or, $\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$

Also $(1-\sin^2\alpha) + (1-\sin^2\beta) + (1-\sin^2\gamma) = 1$

or $\boxed{\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2}$

Important points:-

(i.) A vector can be resolved in maximum infinite numbers of components.

(ii). Maximum numbers of rectangular components of a vector in a plane is two.
But maximum numbers of rectangular components in space (3-D is three.

(iii) Vector addition, when vectors are in terms of rectangular components -

Let two vectors $\vec{A}$ and $\vec{B}$ in three dimensions

as $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

then the resultant vector is

$$\vec{R} = \vec{A} + \vec{B}$$

or $$\boxed{\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}}$$

$$|\vec{R}| = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2 + (A_z + B_z)^2}$$

Note - If $\vec{R} = \vec{A} - \vec{B}$ then

$$\boxed{\vec{R} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j} + (A_z - B_z)\hat{k}}$$

$$|\vec{R}| = \sqrt{(A_x - B_x)^2 + (A_y - B_y)^2 + (A_z - B_z)^2}$$

Multiplication (or) Division of a vector by a scalar:

(i) In multiplication of a vector by a scalar the magnitude becomes K times while the direction remains same.

$$\boxed{\vec{R} = K\vec{A}}$$

(ii) In division of a vector by a scalar, the magnitude becomes 1/K times and the direction remains same.

$$\boxed{\vec{R} = \frac{\vec{A}}{K}}$$

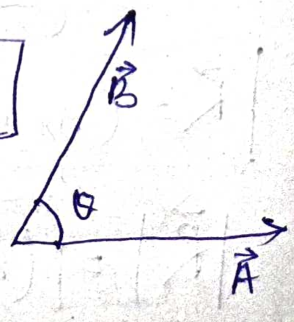
Note:- A scalar (or) a vector, can not be devided by a vector.

Scalar (or) dot product of two vectors:

The scalar product (or dot product) of two vectors is defined as the product of their magnitudes with cosine of the angle between them.

Thus if there are two vectors $\vec{A}$ and $\vec{B}$ having angle θ between them then their scalar product is written as

$$\boxed{\vec{A}\cdot\vec{B} = AB\cos\theta}$$



(or) $$\boxed{\theta = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)}$$

Important Points :-

(i) Scalar product of two vectors is always a scalar. It is positive if angle between the vectors is acute ($\theta < 90°$) and negative if angle between them is obtuse ($\theta > 90°$).

(ii) Scalar product is commutative.

$$\boxed{\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}}$$

(iii) Scalar product is distributive.

$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$$

(iv) Scalar product of two vectors will be maximum when $\cos\theta = \max = 1$

i.e. $\theta = 0°$

i.e. vectors are parallel.

$$\boxed{(\vec{A}\cdot\vec{B})_{max} = AB}$$

(v) Scalar product of two vectors will be zero when $\cos\theta = 0$ i.e. $\theta = 90°$ (vectors are perpendicular to each other).

(vi) For orthogonal unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$

$$\boxed{\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 1\times1\times\cos 90° = 0}$$

and $$\boxed{\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1\times1\times\cos 0° = 1}$$

(vii) In terms of components

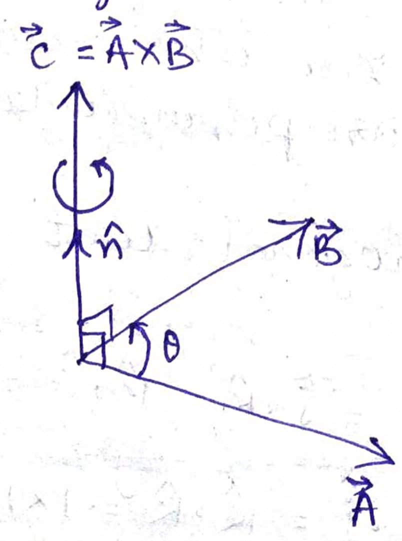
$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

or $\boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z}$

(viii) We can not divide a vector by another vector.

Vector Product of two vectors :-

The vector product (or) cross product of two vectors is defined as a vector having magnitude equal to the product of their magnitudes with the sine of angle between them. and its direction is perpendicular to the plane containing both the vectors according to right hand screw rule or right hand thumb rule.



If $\vec{A}$ and $\vec{B}$ are two vectors, then their vectors product i.e. $\vec{A}\times\vec{B}$ is a vector $\vec{c}$ defined by

$$\boxed{\vec{C} = \vec{A}\times\vec{B} = AB \sin\theta \, \hat{n}}$$

Right Hand Thumb Rule:-

First of all place the vector $\vec{A}$ and $\vec{B}$ tail to tail. Now place stretched fingers and thumb of right hand perpendicular to the plane of $\vec{A}$ and $\vec{B}$ such that the fingers are along the vector $\vec{A}$. If the fingers are now closed through smaller angle so as to go towards $\vec{B}$, then the thumb gives the direction of $\vec{A}\times\vec{B}$ i.e $\vec{c}$.

Examples of vector product:-

(i) Torque $\vec{\tau} = \vec{r}\times\vec{F}$

(ii) Velocity $\vec{v} = \vec{\omega}\times\vec{r}$

(iii) Angular Momentum $\vec{J} = \vec{r}\times\vec{P}$

(iv) Acceleration $\vec{a} = \vec{\alpha}\times\vec{r}$

Here $\vec{r}$ is position vector and $\vec{F}, \vec{P}, \vec{\omega}$ and $\vec{\alpha}$ are Force, linear momentum, angular velocity and angular acceleration respectively.

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Important Points:-

(i) Vector Product of two vectors is not commutative.

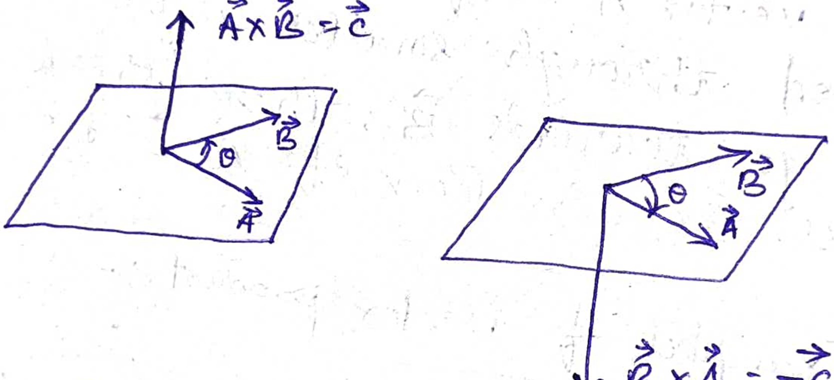
$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

But $|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}| = AB\sin\theta$

Note:-

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

i.e. in case of vectors $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ magnitude are equal but directions are opposite.



(ii) The vector Product is distributive when the order of the vectors is strictly ~~magnit~~ maintained.

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

(iii) The vector product of two vectors will be maximum when $\sin\theta = \max. = 1$

i.e. $\theta = 90^\circ$

i.e. Vector Product is maximum if the vectors are orthogonal (perpendicular)

(iv) The vector Product of two non zero vectors is zero when $|\sin\theta| = 0$
i.e. $\theta = 0°$ or $180°$
i.e. If the vector product of two non-zero vectors is zero, then the vectors are collinear.

(v) In case of orthogonal unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$.

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

and $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$

and $\hat{j} \times \hat{i} = -\hat{k}$, $\hat{k} \times \hat{j} = -\hat{i}$, $\hat{i} \times \hat{k} = -\hat{j}$

(vi) In terms of components

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

or $\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} - (A_x B_z - A_z B_x)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$

(vii) If $\vec{A}$, $\vec{B}$ and $\vec{C}$ are coplanar, then

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = 0$$

(viii) vector product of two vectors is always a vector perpendicular to the plane containing the two vectors.

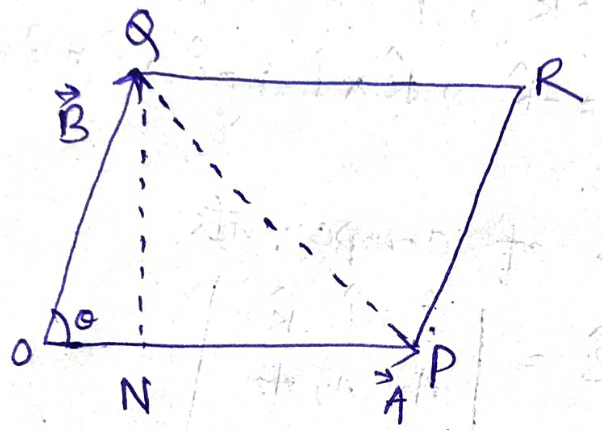
(ix) unit vector Perpendicular to $\vec{A}$ as well as $\vec{B}$ is

$$\boxed{\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}}$$

(x) If $\vec{A}+\vec{B}=\vec{C}$ (or) $\vec{A}+\vec{B}+\vec{C}=0$, then $\vec{A}$, $\vec{B}$ and $\vec{C}$ lie in one plane.

(xi) Projection of a vector $\vec{A}$ in the direction of vector $\vec{B}$ is $= \dfrac{\vec{A}\cdot\vec{B}}{|\vec{B}|}$

Geometrical Meaning of vector Product of two vectors :-



Let two vectors $\vec{A}$ and $\vec{B}$ which are represented by $\vec{OP}$ and $\vec{OQ}$ and $\angle POQ=\theta$

Now complete the parallelogram OPRQ, Join P and Q. here OP = A and OQ = B, Draw $QN \perp OP$.

(i) Magnitude of cross product of $\vec{A}$ and $\vec{B}$

$$|\vec{A}\times\vec{B}| = AB\sin\theta = (OP)(OQ)\sin\theta$$

Or $|\vec{A}\times\vec{B}| = (OP)(NQ)$ $(\because NQ = OQ\sin\theta)$

$$\boxed{|\vec{A}\times\vec{B}| = \text{base}\times\text{height} = \text{Area of parallelogram OPRQ}}$$

★ If $\vec{A}$ and $\vec{B}$ are two adjacent sides of a Parallelogram then its area $= |\vec{A} \times \vec{B}|$

(ii) Area of $\triangle POQ = \dfrac{\text{base} \times \text{height}}{2}$

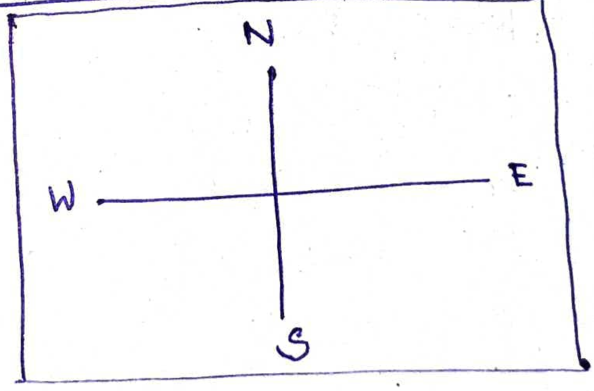
$$= \frac{(OP)(NQ)}{2} = \frac{1}{2}|\vec{A} \times \vec{B}|$$

★ If $\vec{A}$ and $\vec{B}$ are two sides of a triangle, then

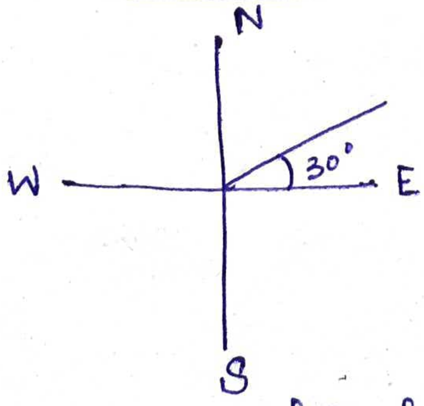
$$\boxed{\text{Area of triangle} = \frac{1}{2}|\vec{A} \times \vec{B}|}$$

(iii) If $\vec{A}$ and $\vec{B}$ are diagonal of a parallelogram then $\boxed{\text{Area of Parallelogram} = \frac{1}{2}|\vec{A} \times \vec{B}|}$

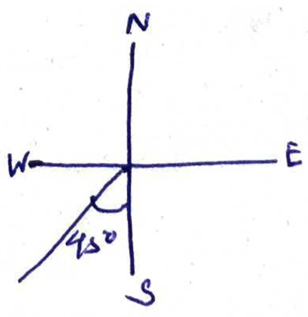
Direction Sense:-



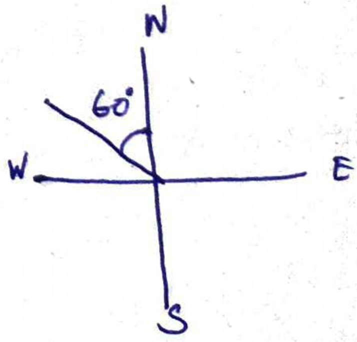
Example :-



30° North of East



45° South-West



60° West of North

Syllabus

Unit : 3 - Kinematics

3.1 Concept of Rest and Motion

3.2 Displacement, speed, velocity, Acceleration and Force.

3.3 Equations of Motion under Gravity.

3.4 Circular Motion : Angular displacement, Angular velocity and Angular acceleration.

3.5 Relation between :
- (i) Linear and Angular velocity
- (ii) Linear and Angular Acceleration

3.6 Define Projectile, Examples of Projectile.

3.7 Expression for Equation of Trajectory, Time of flight, Maximum Height and Horizontal Range for a projectile fired at an angle, Condition for maximum Horizontal Range.

Kinematics

Kinematics:-

It is the study of motion of objects without taking into account the factor which cause the motion.

Motion:-

A body is said to be in motion if it changes its position with time. Motion is always relative to the observer.

Rest :-

If a body does not change its position with time, with respect to its surroundings then it is said to be at rest.

Rectilinear Motion:-

It is a motion in which a particle or point mass body is moving along a straight line.

Point mass object or Particle

If the size of a body is negligible in comparison to its range of motion then the body is said to be point mass object or a particle.

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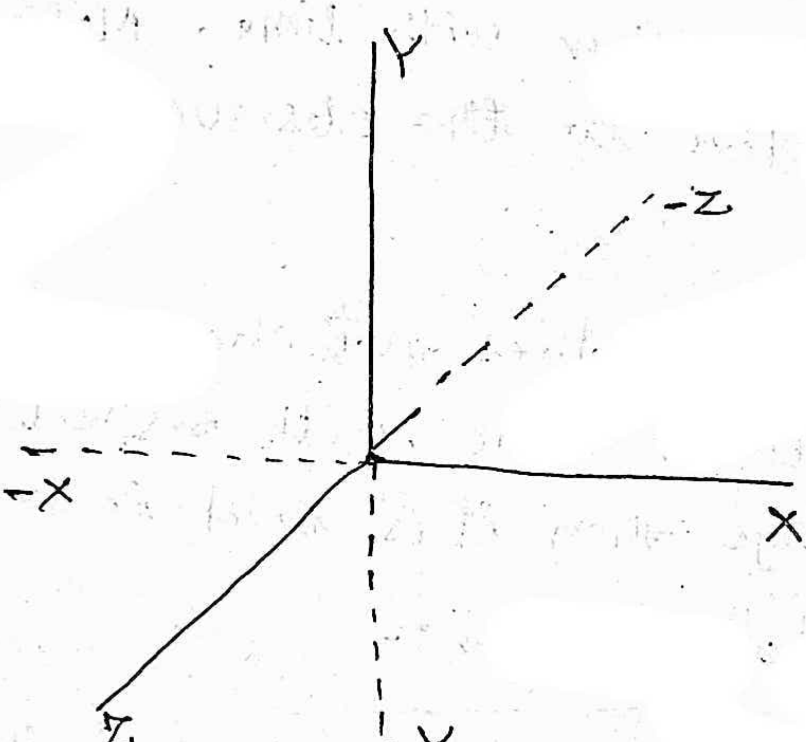
Frame of reference

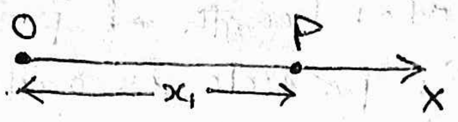
It is a system of three mutually perpendicular axes (x, y and z axis) attached to an observer having a clock with him, w.r.t. which the observer can describe position, displacement, acceleration etc. of a moving object.



The point of intersecting of three axes is called origin, which serves as a reference point or the position of the observer.

If frame is not mention then ground is taken as a reference frame.

1.| One Dimensional Motion :-

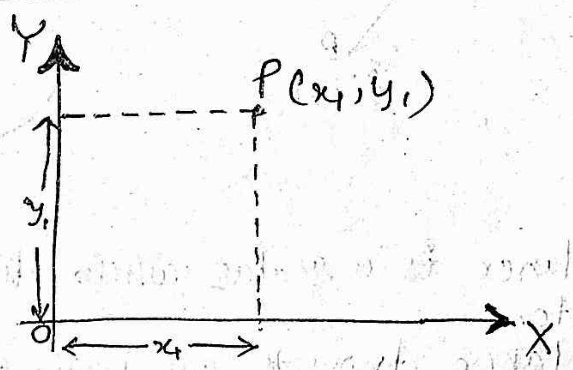


If only one coordinate changes with time, then it is called one dimensional motion.

Example :-

Motion of a car on a straight road, motion of a freely falling body. etc.

2.| Two dimensional Motion :-

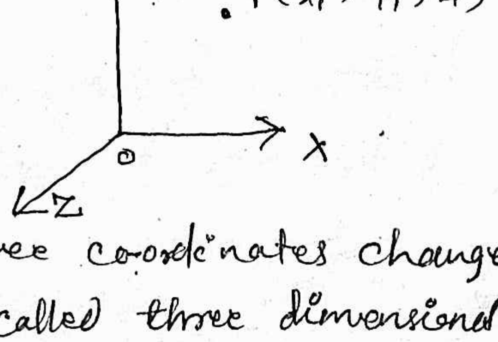


If only two coordinates changes with time, then it is called two dimensional motion.

Example :-

A insect crawling over the floor, motion of a car on circular turn etc.

3.| Three dimensional Motion :-



If all three coordinates changes with time, then it is called three dimensional motion.

Example :-

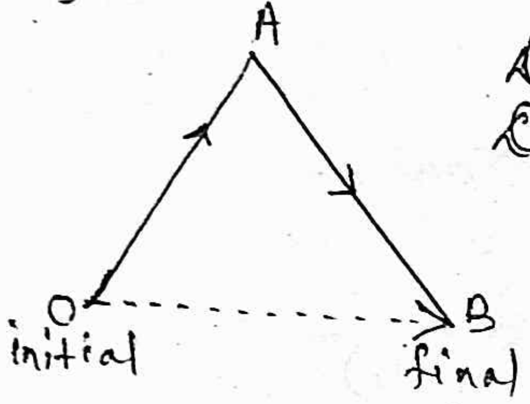
A flying bird, random motion of gas molecule etc.

Distance and Displacement :-

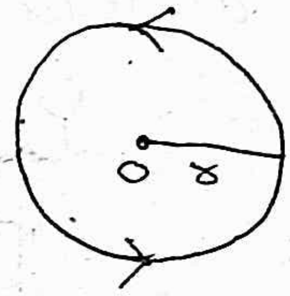
Distance - Total length of path covered by the particle is called the distance.

Displacement -

It is the minimum distance between two positions of the object and its direction is from initial to final position of the object.



Distance = OA + AB
Displacement = OB



Distance = 2πr
Displacement

Note -

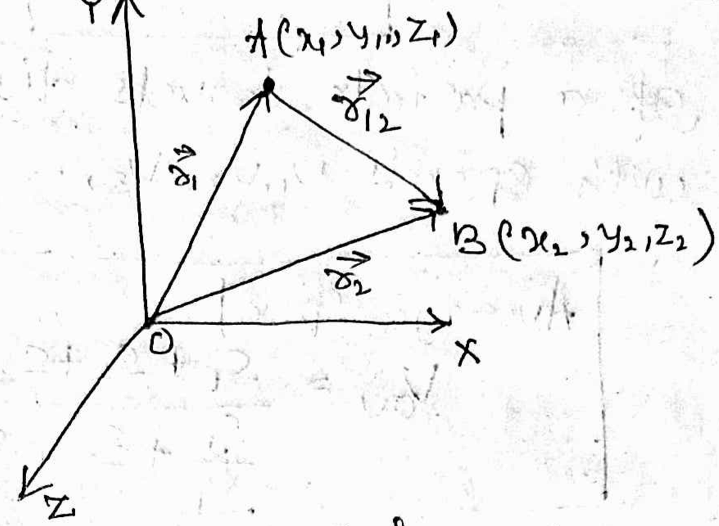
(i) Distance is a scalar while displacement is vector.

(ii) Distance depends on path while displacement does not depends on path, it only depends on initial and final position.

(iii) Distance is always positive or zero while the displacement may be +ve, -ve or zero.

(iv) Distance ≥ |Displacement|

(57) ## Displacement in terms of position vector:-



Let a body is displaced from A (x_1, y_1, z_1) to B (x_2, y_2, z_2) then, its displacement is given by vector $\vec{AB}$.

$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$
$$\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

From ΔOAB $\vec{r}_1 + \vec{r}_{12} = \vec{r}_2$

or $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$

$$\boxed{\begin{aligned}\vec{r}_{12} &= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \\ |\vec{r}_{12}| &= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}\end{aligned}}$$

Speed and Velocity:-

Speed-

It is the rate of distance covered with time. It is related to distance and it is a scalar. The speed of the object can be zero or positive but never negative.

Average speed-

It is defined as the radio of the total distance travelled by the object to the total time taken.

$$\boxed{V_{av} = \frac{\Delta s}{\Delta t}}$$

If a particle travels distances $S_1, S_2, S_3, \ldots$ with speeds $v_1, v_2, v_3, \ldots$ then

Average speed

$$\boxed{V_{av} = \frac{S_1 + S_2 + S_3 + \cdots + S_n}{\frac{S_1}{V_1} + \frac{S_2}{V_2} + \frac{S_3}{V_3} + \cdots + \frac{S_n}{V_n}}}$$

Instantaneous speed -

It is the speed of a particle at a particular instant of time.

If Δt approaches to zero then average speed becomes instantaneous speed.

$$\boxed{V = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}}$$

Velocity -

The rate of change of displacement with time is called velocity.

It is a vector quantity. It can be positive, negative or zero.

Average velocity -

It is defined as the ratio of displacement to the time taken by the body.

$$\boxed{V_{av} = \frac{\Delta \vec{r}}{\Delta t}}$$

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Instantaneous Velocity :-

It is velocity of a particle at a particular instant of time.

$$\boxed{\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}}$$

Acceleration -

The rate of change of velocity with time is called acceleration.

It is a vector quantity. Its direction is same as that of change in velocity.

Average Acceleration -

It is defined as the ratio of total change in velocity to the total time taken.

$$\boxed{a_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}}$$

Instantaneous Acceleration :-

If Δt approaches zero, the average acceleration becomes the instantaneous accelⁿ.

$$\boxed{\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2 \vec{r}}{dt^2}}$$

Note -

Accelⁿ can be +ve, -ve (or) zero.
+ve accelⁿ means velocity increasing with time, zero accelⁿ means velocity is uniform and negative accelⁿ means velocity is decreasing with time.

Equations of kinematics -

First Equation:-

If a particle moves with a constant acceleration $\vec{a}$ then by definition

$$\vec{a} = \frac{d\vec{v}}{dt} \text{ (or) } d\vec{v} = \vec{a}\,dt$$

Let at starting point $(t=0)$ initial velocity is $\vec{u}$ and at time t its final velocity is $\vec{v}$ then -

$$\int_{\vec{u}}^{\vec{v}} d\vec{v} = \int_0^t \vec{a}\,dt \Rightarrow \int_{\vec{u}}^{\vec{v}} d\vec{v} = \vec{a}\int_0^t dt$$

or $[\vec{v}]_{\vec{u}}^{\vec{v}} = \vec{a}[t]_0^t$ or $\vec{v} - \vec{u} = \vec{a}t$

or $\boxed{\vec{V} = \vec{u} + \vec{a}t}$

In scalar form $\boxed{v = u + at}$

Second Equation:-

By definition of velocity $\vec{v} = \frac{d\vec{s}}{dt}$

or $\frac{d\vec{s}}{dt} = \vec{u} + \vec{a}t$

or $d\vec{s} = (\vec{u} + \vec{a}t)\,dt$

let at time t, displacement is $\vec{s}$ then

$$\int_0^{\vec{s}} d\vec{s} = \int_0^t (\vec{u} + \vec{a}t)\,dt$$

$$\boxed{\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2}$$

In scalar form $\boxed{s = ut + \frac{1}{2}at^2}$

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Third Equation:-

We know that $v = u + at$ (or)

$$t = \left(\frac{v-u}{a}\right)$$

by substituting the value of t in eq: $s = ut + \frac{1}{2}at^2$ we get

$$s = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2$$

or $$2as = 2uv - 2u^2 + v^2 + u^2 - 2uv$$

or $$\boxed{v^2 = u^2 + 2as}$$

In vector form $\boxed{v^2 = u^2 + 2\vec{a}\cdot\vec{s}}$

Distance travelled by the body in n^{th} second -

$$S_n = S_n - S_{n-1}$$

$$S_n = \left[un + \frac{1}{2}an^2\right] - \left[u(n-1) - \frac{1}{2}a(n-1)^2\right]$$

on solving we get

$$\boxed{S_n = u + \frac{a}{2}(2n-1)}$$

In vector form $\boxed{\vec{S}_n = \vec{u} + \frac{\vec{a}}{2}(2n-1)}$

Equations of Motion under Gravity:

If air resistance is negligible and a body is freely moving along vertical line near the earth surface then an acceleration acts downward which is $9.8\ m/s^2$.

Note

(i) velocity - time relation, $v = u + gt$

(ii) Displacement - time relation $h = ut + \frac{1}{2}g$

(iii) velocity displacement relation, $v^2 - u^2 = 2gh$

(iv) Displacement in nth second $S_{nth} = u + \frac{g}{2}(2n-1$

1. If a body is dropped from some height (u=

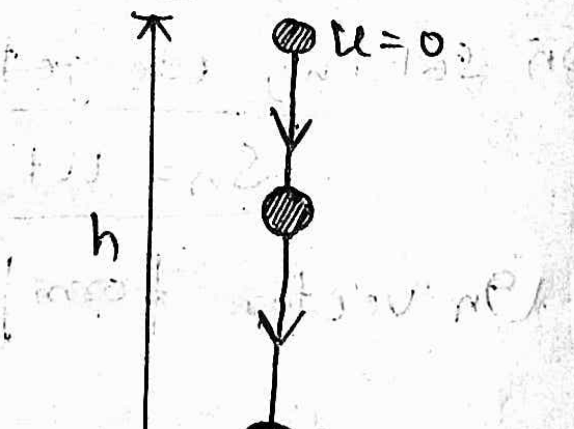
In this case, $u = 0$, $a = +g$ then equations of motion are -

(i) $v = gt$

(ii) $h = \frac{1}{2}gt^2$

(iii) $v^2 = 2gh$

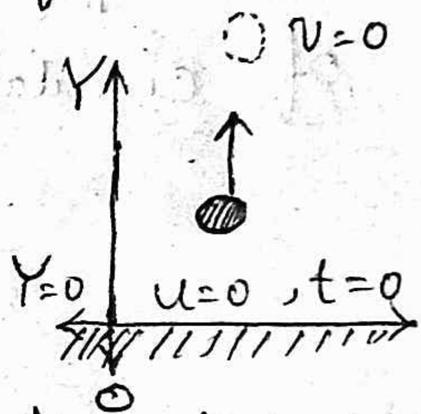
(iv) $h_n = \frac{g}{2}(2n-1)$



2.] If a body is projected vertically upward-

Taking initial position as origin and direction of motion (i.e. vertically up) as positive y axis.

$v=0$ at maximum height at $t=T$ and $a=-g$ then equations of motion are



$0 = u - gt$ (or) (i) $u = gt$

(ii) $h = ut - gt^2$

or $h_{max} = uT - \frac{1}{2}gT^2 \Rightarrow h_{max} = (gT)T - \frac{1}{2}gT^2$

or $\boxed{h_{max} = \frac{1}{2}gT^2}$

(iii) $v^2 = u^2 - 2gh$ (or) $0 = u^2 - 2gh_{max}$

or $\boxed{u^2 = 2gh_{max}}$

(iv) $S_{nth} = u + \frac{a}{2}(2n-1) = u - \frac{g}{2}(2n-1)$

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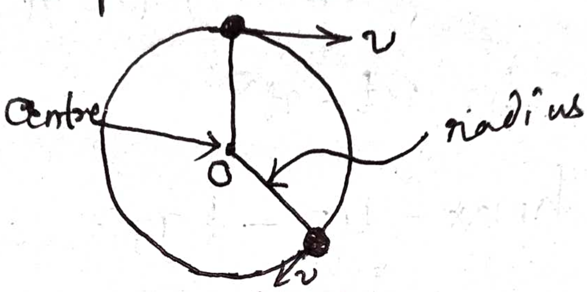
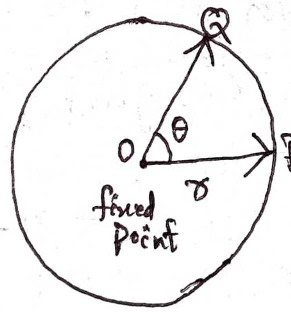
Circular Motion

When a particle moves in a plane such that its distance from a fixed point remains constant then its motion is called as circular motion respect to the fixed point.

The fixed point is called centre and the distance is called radius of circular path.



Angular Displacement (θ) -



Angle traced by position vector of a particle moving w.r.t. some fixed point is called angular displacement.

$$\boxed{\text{Angular displacement } \theta = \frac{\text{Arc } PQ}{r}}$$

★ Its direction is perpendicular to plane of rotation and given by right hand screw rule.

★ It is dimensionless and has SI unit 'Radian'

2π radian $= 360^\circ = 1$ revolution.

<u>Note</u> -

Small angular displacement $d\vec{\theta}$ is a vector quantity.

But large angular displacement θ is scalar quantity.

<u>Frequency (n)</u> -

Numbers of revolutions describes by particle per second is its frequency.

Its unit is <u>revolution per sec.</u> (r.p.s.)

<u>Time Period (T)</u> -

It is time taken by particle to complete one revolution.

$$\boxed{T = \frac{1}{n}}$$

Angular velocity (ω) -

It is defined as the rate of change of angular displacement of moving object. $\boxed{\omega = \frac{\theta}{t}}$,

$$\text{Angular velocity} = \frac{\text{Angle traced}}{\text{Time taken}}$$

$$\boxed{\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}}$$

★ Its SI unit is radian/sec.

★ It is an axial vector quantity. Its direction is perpendicular to the plane of rotation and along the axis according to right hand screw rule.

Relation between linear and Angular velocity:-

$$\text{Angle} = \frac{\text{Arc}}{\text{Radius}} \quad \text{or} \quad \Delta\theta = \frac{\Delta s}{r}$$

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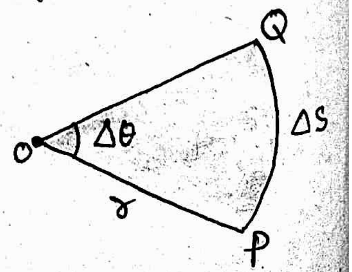
or $\Delta s = r\Delta\theta$

$\therefore \frac{\Delta s}{\Delta t} = \frac{r\Delta\theta}{\Delta t}$ if $\Delta t \to 0$ then $\frac{ds}{dt} = r\frac{d\theta}{dt}$



Or $\boxed{v = r\omega}$

In vector form $\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$

direction of $\vec{v}$ is according to right hand thumb rule.

Average Angular Velocity (ω_{av}) -

$$\omega_{av} = \frac{\text{total angle of rotation}}{\text{total time taken}}$$

$$\boxed{\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}}$$

where θ_1 and θ_2 are angular position of the particle at instant t_1 and t_2.

Instantaneous Angular Velocity (ω)-

It is the angular velocity at a particular instant.

$$\boxed{\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}}$$

Or $$\boxed{\vec{\omega} = \frac{d\vec{\theta}}{dt}}$$

Angular Acceleration (α)

The rate of change of angular velocity is called angular acceleration.

$$\boxed{\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}}$$

Or $$\boxed{\vec{\alpha} = \frac{d\vec{\omega}}{dt}}$$

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* It is an axial vector quantity. Its direction is along the axis according to the 'Right hand Rule'.
* Its SI unit is radian/sec².

Relation between Angular and linear Acceleration :-

We know that $\vec{v} = \vec{\omega} \times \vec{r}$

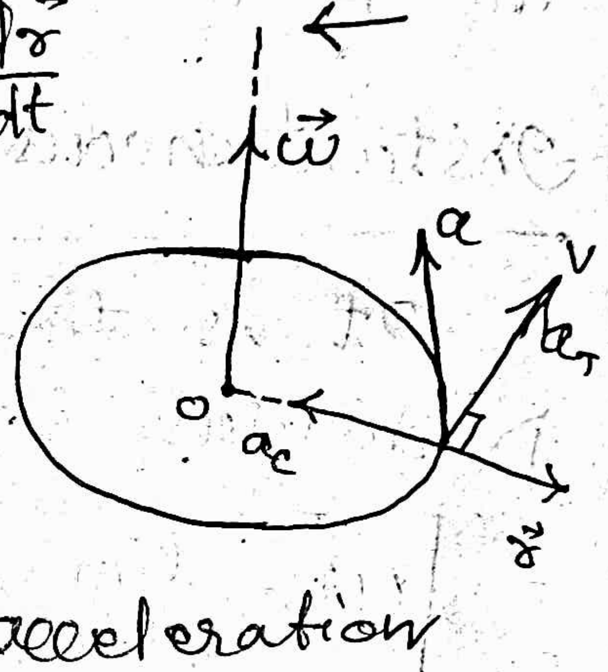
Here $\vec{v}$ is a tangential vector, $\vec{\omega}$ is a axial vector and $\vec{r}$ is a radial vector. These three vectors are mutually perpendicular.

but $\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow \vec{a} = \frac{d}{dt}(\vec{\omega} \times \vec{r})$

or $\vec{a} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$

$$\boxed{\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}}$$

$$\boxed{\vec{a} = \vec{a}_T + \vec{a}_c}$$



Here $\vec{a}_T$ is tangential acceleration and $\vec{a}_c$ is centripetal acceleration.

> Hence $\vec{a}_T$ and $\vec{a}_c$ are two components of net linear acceleration.

(69)

Tangential Acceleration ($\vec{a}_T$) -

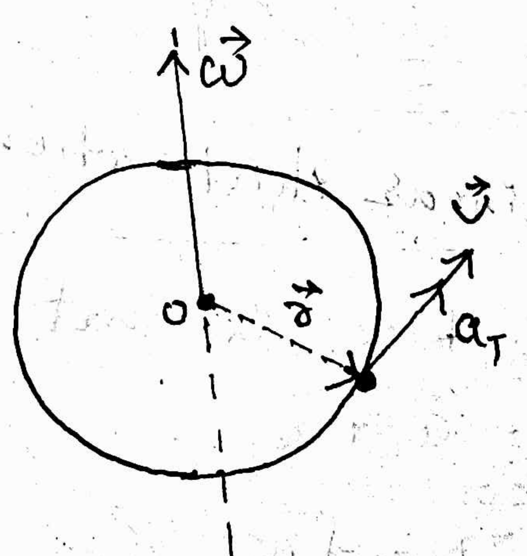
$\vec{a}_T = \vec{\alpha} \times \vec{r}$, its direction is parallel to velocity.

As $\vec{\omega}$ and $\vec{\alpha}$ both are parallel and along the axis so that $\vec{v}$ and $\vec{a}_T$ are also parallel and along the tangential direction.

Magnitude of tangential acceleration is $a_T = \alpha r \sin 90^\circ$

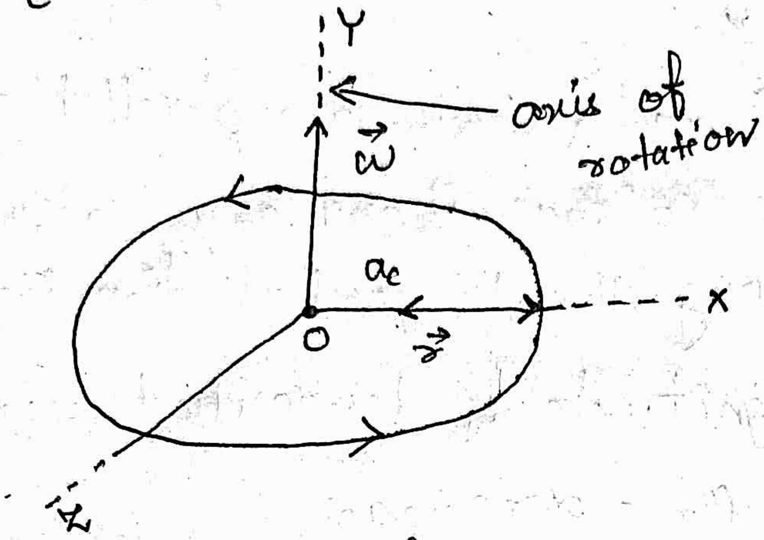
$$\Rightarrow \boxed{a_T = \alpha r}$$

As $\vec{a}_T$ is along the direction of motion (along $\vec{v}$) so that $\vec{a}_T$ is responsible for change in speed of particle. Its magnitude is rate of change of speed of the particle. On circular path with constant speed tangential acceleration is zero.



Centripetal Acceleration (a_c)

$$\vec{a}_c = \vec{\omega} \times \vec{v} \Rightarrow \vec{a}_c = \vec{\omega} \times (\vec{\omega} \times \vec{r})$$



Let $\vec{r}$ is in $\hat{i}$ direction and $\vec{\omega}$ is in $\hat{j}$ direction, then direction of $\vec{a}_c$ is along $\hat{j} \times (\hat{j} \times \hat{i}) = \hat{j} \times (-\hat{k}) = -\hat{i}$

opposite direction of $\vec{r}$ i.e. from P to O and it is centripetal direction.

Magnitude of centripetal acceleration is

$$\boxed{a_c = \omega v = \frac{v^2}{r} = \omega^2 r}$$

$$\Rightarrow \boxed{\vec{a}_c = \frac{v^2}{r}(-\hat{r})}$$

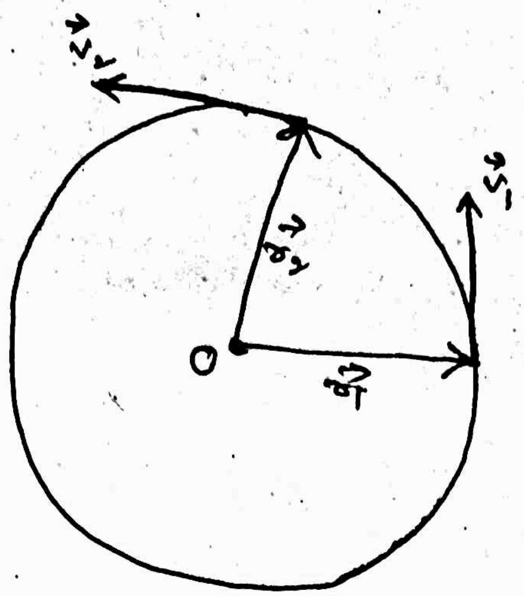
Net Linear Acceleration :-

As $a_T \perp a_c$ so that

by $\boxed{\vec{a} = \vec{a}_T + \vec{a}_c}$

$$\Rightarrow \boxed{|\vec{a}| = \sqrt{a_T^2 + a_c^2}}$$

(II) Uniform Circular Motion :-



When a particle moves in a circle at a constant speed then the motion is said to be a uniform circular motion.

On this motion, position vector keep changing continuously.

Speed is constant, so that

$\boxed{\vec{a}_T = 0}$ $\boxed{\alpha = 0}$

Acceleration of particle $\vec{a} = \vec{a}_c = \vec{\omega} \times \vec{v}$

or $a = \omega v$

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$\therefore \boxed{a = \frac{v^2}{r} = \omega^2 r = \text{Centripetal acceleration}}$

Due to centripetal acceleration the velocity of the particle keeps on changing the direction i.e. the particle is accelerated.

(13) # Two dimensional Motion :-

If two co-ordinates are required to specify the position of the object in space changes w.r.t. time then it is called two dimensional motion.

In such a motion, the object moves in a plane.

Projectile Motion:-

A particle thrown in the space which moves under the effect of gravity alone is called projectile and its motion is called 'projectile motion'.

Example:-

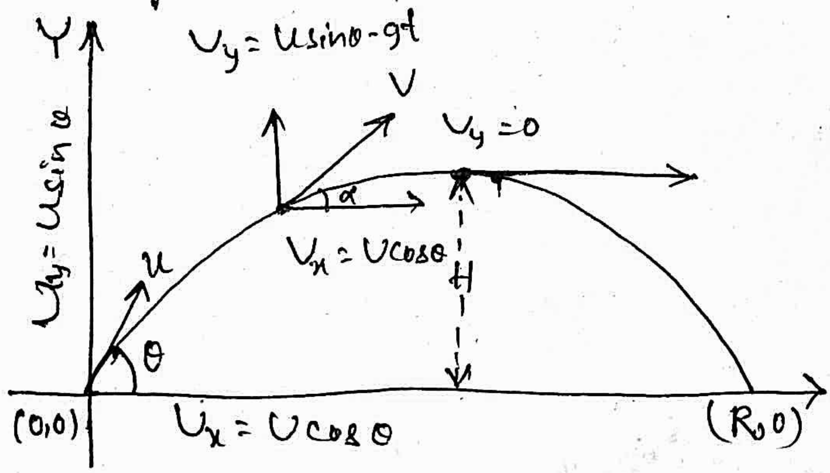
(i) A bomb released from an aeroplane

(ii) An arrow released from bow.

Note:-

(i) Projectile motion is an example of constant acceleration.

(ii) If a constant acceleration is given to particle in oblique direction with initial velocity, then the resultant path is parabolic



Ground to Ground projection :-

The projectile motion can be considered as two mutually perpendicular motion, which are independent of each other.

i.e. Projectile motion =

Horizontal Motion + Verticle motion

Horizontal Motion :-

→ Initial Velocity $u_x = u\cos\theta$

→ Acceleration $a_x = 0$ (Neglect air resistance)

Therefore horizontal velocity remains unchanged.

~~Horizontal~~ velocity at any instant

$$u_x = u\cos\theta$$

→ At any time t, x co-ordinate or displacement along x-direction is

$x = u_x t$ or $x = (u\cos\theta) \times t$

Verticle Motion :-

It is the motion under the effect of gravity, so that as particle moves upwords its verticle speed decreases.

i.e. $a_y = -g$

→ verticle speed at time t, $v_y = u_y - gt$

→ Displacement in verticle direction in time t,

$$y = u_y t - \frac{1}{2} g t^2 \quad \text{(or)}$$

$$y = (u\sin\theta) t - \frac{1}{2} g t^2$$

Here 'y' is the 'height' of the particle above the ground.

Net Motion :-

Net initial velocity $\vec{u} = u_x\hat{i} + u_y\hat{j}$

$$\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$$

Net acceleration $\vec{a} = a_x\hat{i} + a_y\hat{j}$

$$\vec{a} = -g\hat{j} \quad [\because a_x = 0]$$

Co-ordinate of particle at time t :-

$$x = u_x t \text{ and } y = u_y t - \frac{1}{2}gt^2$$

Net displacement in time t is

$$\boxed{S = \sqrt{x^2 + y^2}}$$

Velocity of particle at time t :-

$$\vec{V} = V_x\hat{i} + V_y\hat{j} \Rightarrow \vec{V} = u_x\hat{i} + (u_y - gt)\hat{j}$$

Or $\vec{V} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}$

Magnitude of velocity $\boxed{V = \sqrt{V_x^2 + V_y^2}}$

If angle of velocity $\vec{v}$ from the ground is α, then

$$\boxed{\tan\alpha = \frac{V_y}{V_x}} \quad \tan\alpha = \frac{u\sin\theta - gt}{u\cos\theta}$$

Time of flight (T) -

At time $t = T$, the particle will be at ground again i.e. displacement along y-axis becomes zero.

$\because \; y = (u\sin\theta)t - \frac{1}{2}gt^2$

$\therefore \; 0 = (u\sin\theta)T - \frac{1}{2}gT^2$

Or $\boxed{T = \frac{2u\sin\theta}{g}}$ or, $\boxed{T = \frac{2u_y}{g}}$

$$\boxed{\text{Assending time} = \text{Desending time} = \frac{T}{2} = \frac{u\sin\theta}{g}}$$

At time $\frac{T}{2}$, Particle attains maximum height of its trajectory.

Horizontal Range (R) -

It is the displacement of the particle along x-direction during its complete flight.

$\because \; x = (u\cos\theta)t \qquad \therefore \; R = (u\cos\theta)T$

Or $R = u\cos\theta \times \frac{2u\sin\theta}{g}$

$= \frac{2u^2\sin\theta\cos\theta}{g}$

$\boxed{R = \frac{u^2\sin 2\theta}{g}} \Rightarrow \boxed{R = \frac{2u_x u_y}{g}}$

Maximum Height attained (H)-

At maximum height verticle velocity becomes zero.

At this instant y coordinate is its maximum height.

$$\because V_y^2 = u_y^2 - 2gy$$

$$\therefore 0 = u_y^2 - 2gH$$

$$H = \frac{u_y^2}{2g} \Rightarrow \boxed{H = \frac{u^2 \sin^2\theta}{2g}}$$

$$\boxed{H = \frac{u_y^2}{2g}}$$

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Equation of Trajectory -

We know that $x = (u\cos\theta)t$

and $y = (u\sin\theta)t - \frac{1}{2}gt^2$

on eliminating it from these two equations, we get

$$y = (u\sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

or $$\boxed{y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}}$$

This is the equation of Parabola.

$$[\, y = ax - bx^2 \,]$$

Energy of Projectile -

When a Projectile moves upward its kinetic energy decreases,

potential energy increases but the total energy always remains constant.

Kinetic energy $= \frac{1}{2}m(u\cos\theta)^2 = \frac{1}{2}mu^2\cos^2\theta$

Potential energy $= mgH = mg\dfrac{u^2\sin^2\theta}{2g}$

$= \frac{1}{2}mu^2\sin^2\theta$

Total Energy = kinetic energy + Potential energy

$= \frac{1}{2}mu^2\cos^2\theta + \frac{1}{2}mu^2\sin^2\theta$

$= \frac{1}{2}mu^2 =$ Energy at the Point of projection

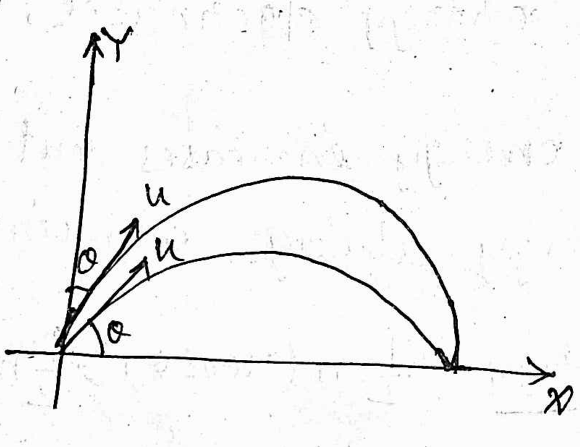
This is in accordance with the law of conservation of energy.

Note :-

1.] Magnitude of velocity at any height 'h' by energy conservation Law $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgh$

Or $\boxed{V = \sqrt{u^2 - 2gh}}$

2.] When two projectiles are thrown with equal speed at angle θ and $(90^\circ - \theta)$ then their ranges are equal but maximum height attained are different and time of flight are also different.



3.] For maximum Range $\theta = 45^\circ$

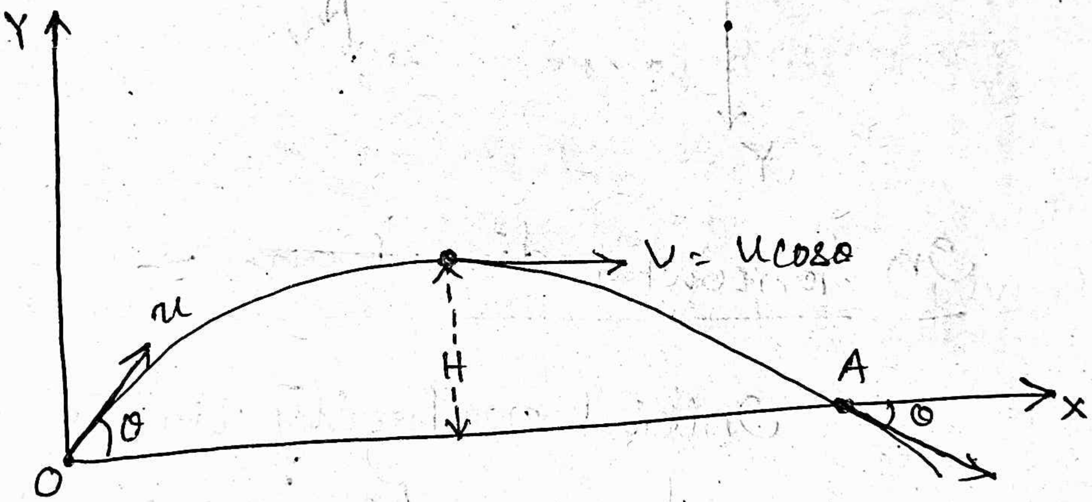
and $R_{max} = \frac{u^2 \sin 2(45^\circ)}{g}$

$\Rightarrow \boxed{R_{max} = \frac{u^2}{g}}$

4.] At maximum height angle between velocity and acceleration is 90°.

5.] When particle returns to ground at point A, then its y-coordinate is zero and its magnitude of velocity is u at angle θ with ground.

Hence total angular change $= 2\theta$



Initial Velocity $\vec{u}_i = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

Final Velocity $\vec{u}_f = u\cos\theta\,\hat{i} - u\sin\theta\,\hat{j}$

Total change in velocity $= |\Delta\vec{v}|$

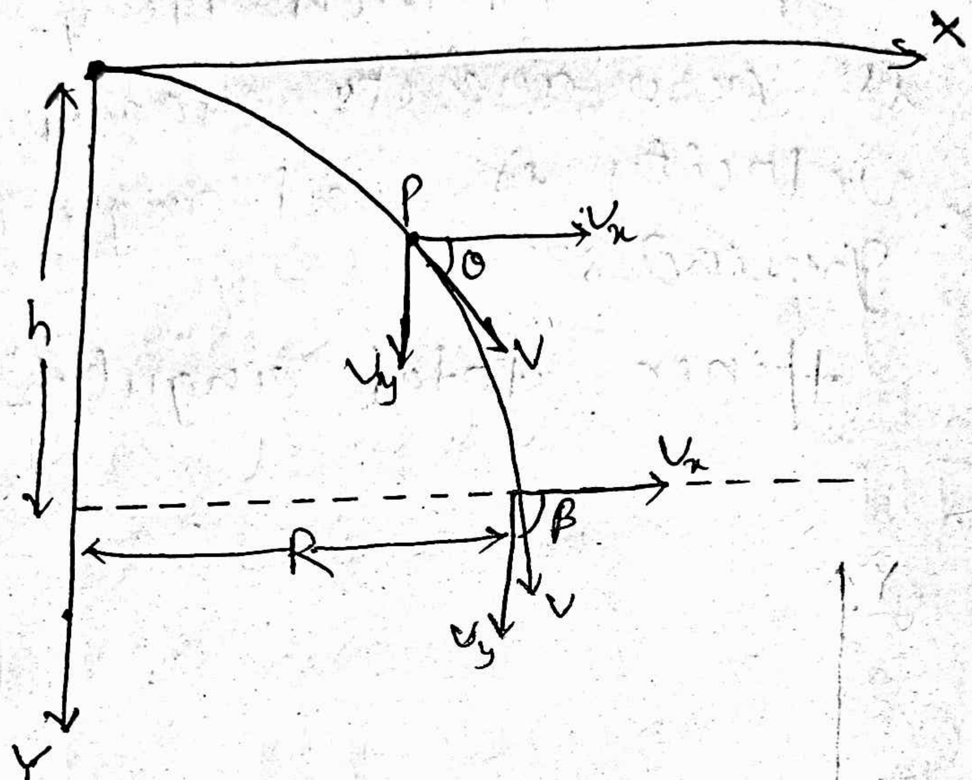
$= 2u\sin\theta$

Total change in momentum

$= m|\Delta\vec{v}| = 2mu\sin\theta$

Horizontal Projectile Motion :-

Consider a Projectile thrown from point O at some height h from the ground with velocity u.



In Horizontal direction :-

Initial velocity $u_x = u$

Acceleration $a_x = 0$

In vertical direction :-

Initial velocity $u_y = 0$

Acceleration $a_y = -g$

Trajectory Equation :-

The path traced by projectile is called the trajectory.

After time t, $x = ut$ and $y = -\frac{1}{2}gt^2$

Here -ve singn indicates that the direction of verticle displacement is in downward direction.

So that $\boxed{y = -\frac{1}{2} g \cdot \frac{x^2}{u^2}}$

$\left[\because t = \frac{x}{u}\right]$

This is the equation of parabola.

Instantaneous velocity at Point P(x,y)-

Horizontal velocity at time t,

$v_x = u$ [remains const.]

Vertical velocity at time t,

$v_y = gt$ [downward]

$\therefore \boxed{v = \sqrt{v_x^2 + v_y^2}} \Rightarrow \boxed{v = \sqrt{u^2 + g^2t^2}}$

and $\boxed{\tan\theta = \frac{v_y}{v_x}}$ or $\boxed{\tan\theta = \frac{-gt}{u}}$

Here -ve sign indicates clockwise direction.

Displacement :-

$$\vec{s} = x\hat{i} + y\hat{j}$$

where $\boxed{|\vec{s}| = \sqrt{x^2+y^2}}$

or $\boxed{\vec{s} = (ut)\hat{i} - (\frac{1}{2}gt^2)\hat{j}}$

Time to flight -

Time taken by the particle to reach the ground.

From equation of motion for vertical direction.

$$h = u_y t + \frac{1}{2}gt^2$$

At heighest point $u_y = 0$

$$h = \frac{1}{2}gT^2$$

or $\boxed{T = \sqrt{\frac{2h}{g}}}$

Horizontal Range :-

Distance covered by the projectile along the horizontal direction between the point of projection to the point on the ground.

$$R = u_x t \Rightarrow \boxed{R = u\sqrt{\frac{2h}{g}}}$$

velocity after falling height h :-

Along verticle direction

$v_y^2 = 0 + 2gh$

or $v_y = \sqrt{2gh}$

Along horizontal direction $v_x = u$

Hence Final velocity

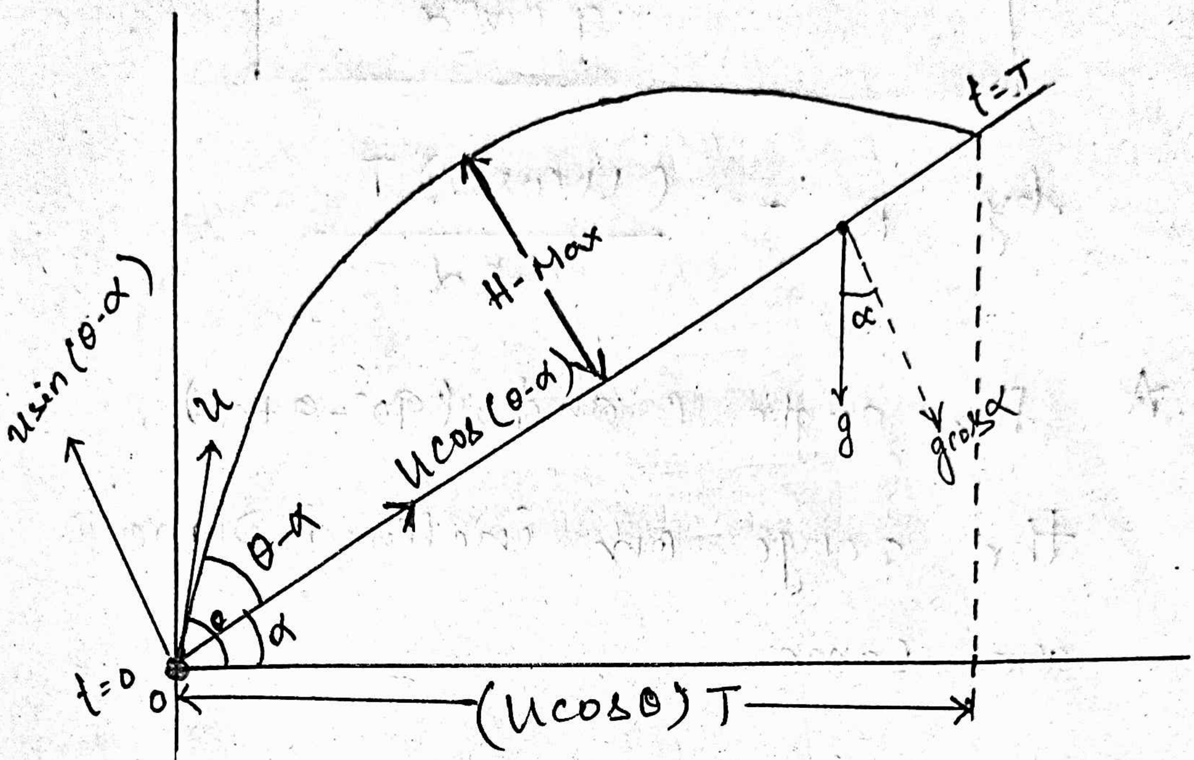
$$\boxed{v = \sqrt{v_x^2 + v_y^2}}$$

$$\Rightarrow \boxed{v = \sqrt{u^2 + 2gh}}$$

and $$\boxed{\tan\beta = \frac{v_y}{v_x}}$$

$$\Rightarrow \boxed{\tan\beta = \frac{\sqrt{2gh}}{u}}$$

Projectile Motion on inclined plane - up Motion



→ Time of flight (T)

$$\boxed{T = \frac{2u\sin(\theta-\alpha)}{g\cos\alpha}}$$

→ Maximum height (Hmax)

$$\boxed{H_{max} = \frac{u^2\sin^2(\theta-\alpha)}{2g\cos\alpha}}$$

→ Range on inclined Plane (R)

$$\boxed{R = \frac{2u^2 \sin(\theta - \alpha)\cos\theta}{g\cos^2\alpha}}$$

by $R = \dfrac{(u\cos\theta)T}{\cos\alpha}$

★ For angle θ and $(90° - \theta + \alpha)$, the range on inclined plane are same.

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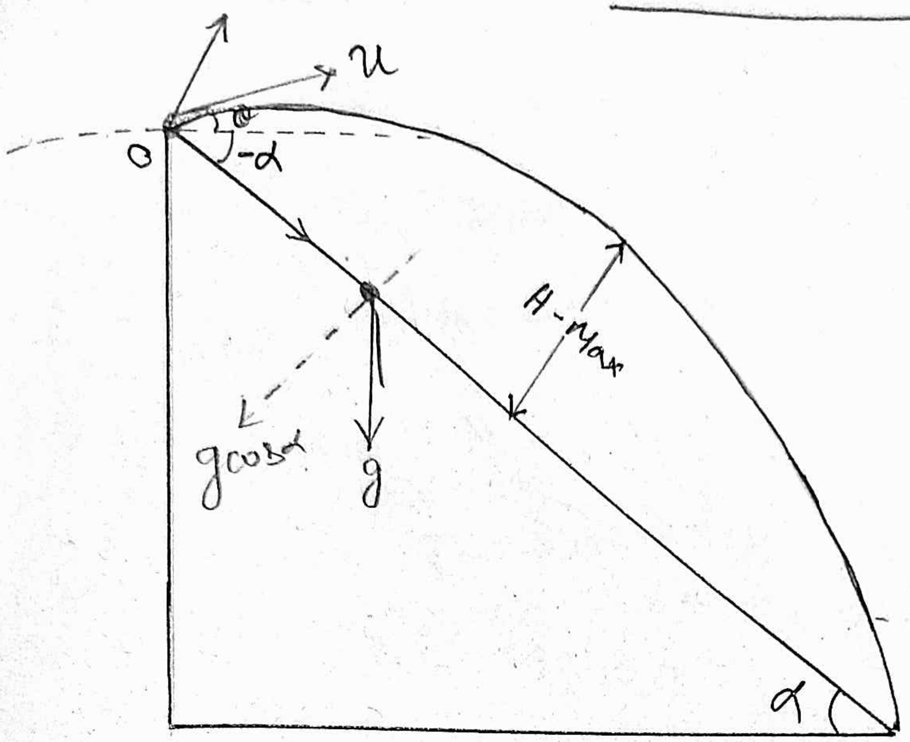
Note -

For maximum Range on inclined Plane $\theta = \frac{\pi}{4} + \frac{\alpha}{2}$

Projectile Motion on inclined plane - down Motion



→ Time of Flight

$$T = \frac{2u\sin(\theta+\alpha)}{g\cos\alpha}$$

→ Maximum Height

$$H = \frac{u^2\sin^2(\theta+\alpha)}{2g\cos\alpha}$$

→ Range on inclined Plane

$$R = \frac{2u^2\cos\theta\sin(\theta+\alpha)}{g\cos^2\alpha}$$

(Put '$-\alpha$' in place of α in above Eqⁿ)

Newton's Law of Motion

Point Mass -

An object can be considered as a point mass if during motion in a given time, it covers distance much greater than its own size.

Inertia -

It is the property of the body due to which the body opposes the change of itself state.

Inertia of a body is measured by mass of the body.

$$\boxed{\text{Inertia} \propto \text{mass}}$$

Heavier the body, greater is the force required to change its state and hence greater is the inertia.

→ Inertia has no unit and no dimensions.

→ Two body of equal mass, one in motion and another is at rest, has the same inertia because it is a factor of mass only and does not depend upon the body.

Types of inertia

1. Inertia of rest :-

It is the inability of a body to change by itself, its state of rest. This means a body at rest remains at rest and cannot start moving by its own.

eg. A person who is standing freely in bus, thrown backward when bus starts suddenly.

2. Inertia of Motion :-

It is the inability of the body to change by itself its state of uniform motion.

eg. A person jumps out of a moving train may fall forward.

3.) Inertia of direction -

It is the inability of a body to change by itself its direction of Motion.

Eg. When a car goes round a curve suddenly, the person sitting inside it, is thrown outwards.

4.) Inertia of Rotation :-

It is inability of a body to change by itself its rotational axis.

Eg. It keeps balance of moving cycle.

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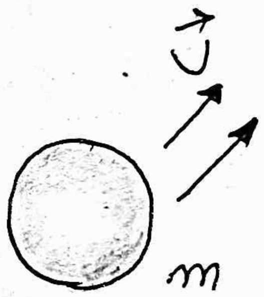
Linear Momentum :-

The total quantity of motion possessed by a moving body is known as the momentum of the body. It is measured as the product of the mass of the body and its velocity.

Momentum = mass × velocity

$$\boxed{\vec{P} = m\vec{v}}$$



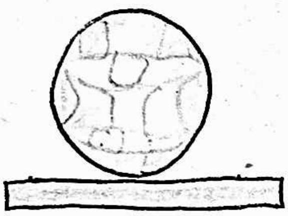
It is a vector quantity and its direction is same as the direction of velocity of the body.

SI unit - Kg. m/sec.

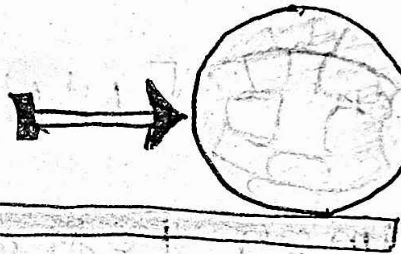
Dimension - $[M^1L^1T^{-1}]$

Newton's First Law -

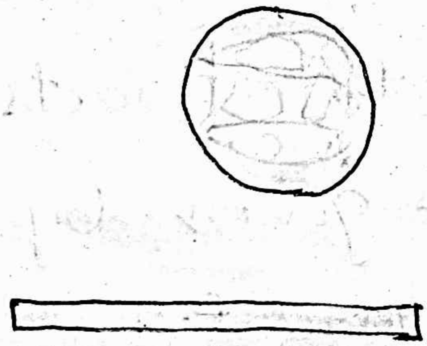
Every body continues in its state of rest or uniform motion along a straight line, unless it is acted upon by some external force to change the state.



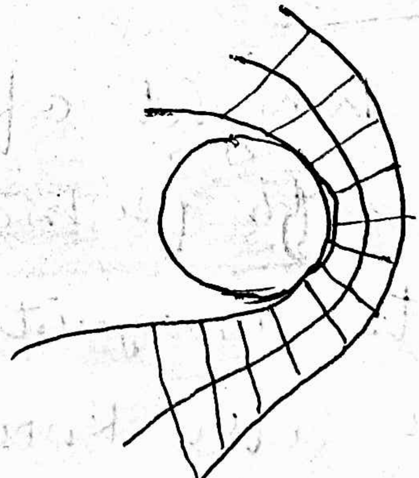
An object at rest will remain at rest...



unless acted on by an unbalanced force.



An object in motion will continue with constant speed and direction...



...unless acted on by an unbalanced force.

Force -

Force is that push or pull which changes or tends to change the state of rest or of uniform motion in a straight line.

SI unit - Newton

Dimensions - $[M^1 L^1 T^{-2}]$

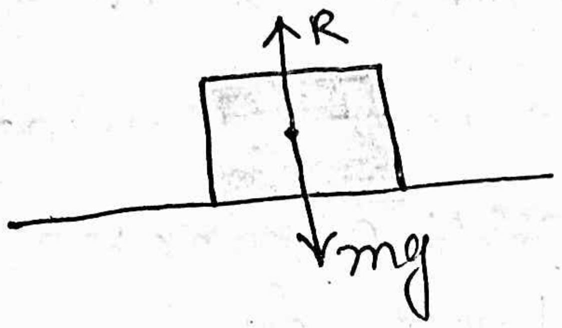
Common Forces in Mechanics

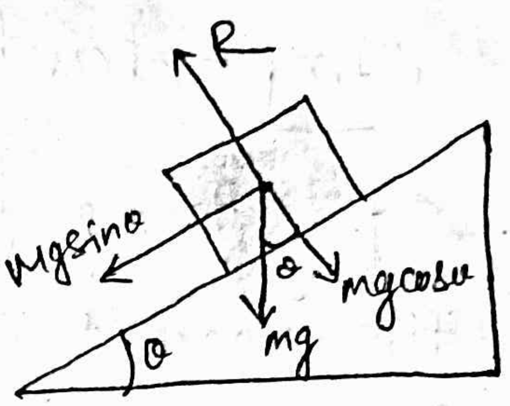
1.) Weight :-

Weight of a body is the force with which earth attracts it. Its magnitude is given by mg. It is also known as gravitational force.

2.) Reaction or Normal Force -

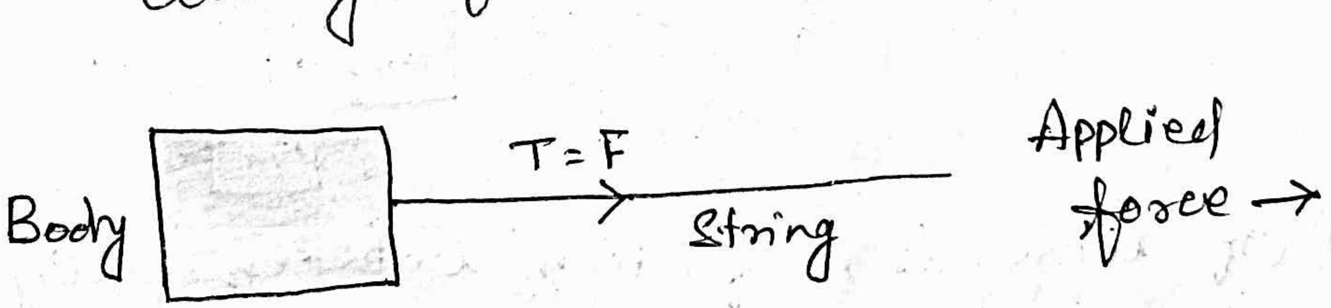
When a body is placed on a rigid surface, then the body experiences a force which is perpendicular to the surface in contact. This force is called reaction or normal force.





3. Tension :-

The force exerted by the end of taut string, rope or chain against pulling (applied) force is called the tension. The direction of tension is away from the body.



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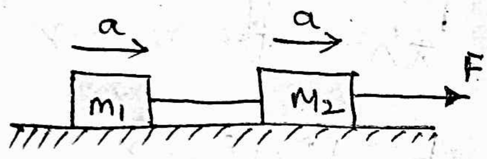
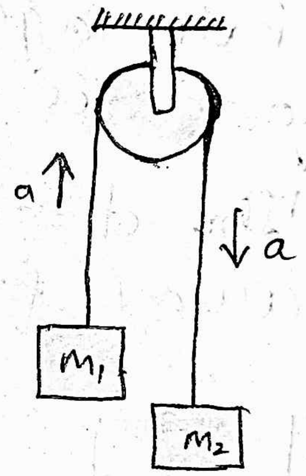
Important Points :-

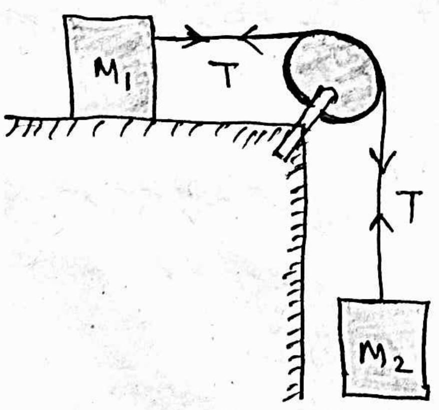
(i) If string is inextensible then the magnitude of acceleration of any number of masses connected through string is always same.

(ii) If string is massless then the tension in it is same everywhere.





(iii) If there is no friction between pully and string then the tension will be same on both sides of pully.



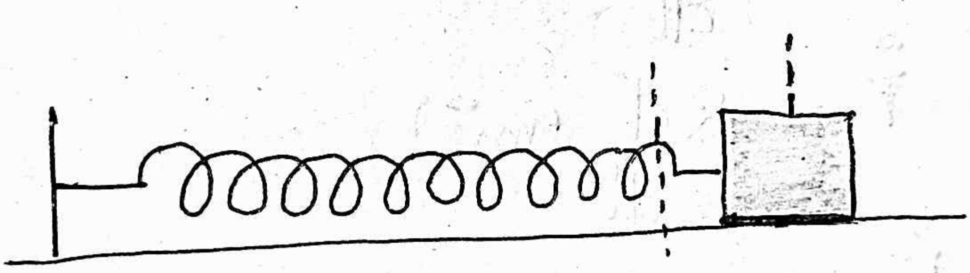
Note-
If there is friction between string and pully then, tension will be different on both sides of pully.

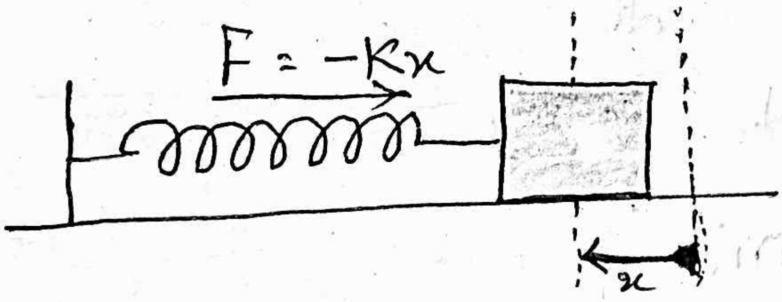
4. ## Spring Force -

Every spring resists any change in its length. This resistive force increases with change in length. It is given by -

$$\boxed{F = -Kx}$$

Here x is the change in length and K is the spring constant.





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Newton's Second Law-

The rate of change of linear momentum of a body is directly proportional to the external force applied on it and this change in momentum takes place in the direction of the applied force.

Hence $F \propto \frac{d\vec{P}}{dt}$

or, $F \propto \frac{d}{dt}(m\vec{v})$

or, $\vec{F} = k\frac{d}{dt}(m\vec{v})$

$\therefore$ $\vec{F} = k \cdot m \frac{d(\vec{v})}{dt}$

If $k = 1$

$\Rightarrow \boxed{\vec{F} = m\vec{a}}$

* Newton's second law gives the quantitative definition of force.

→ Applications of Newton's 2nd law of Motion :-

▷ A cricketer lowers his hands while catching the ball

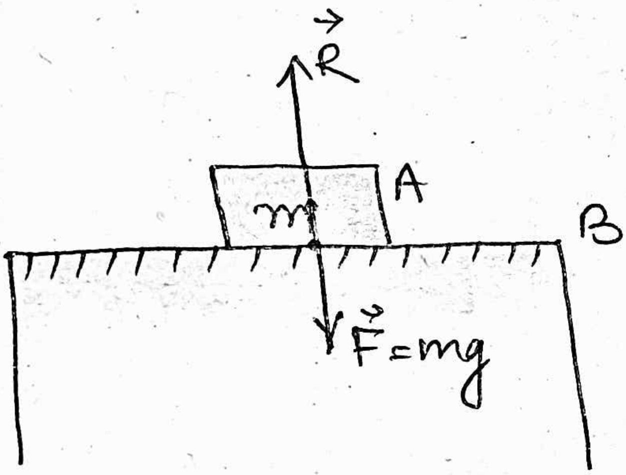
$$\boxed{F = \frac{m(v-u)}{t}}$$

Newton's Third law -

To every action, there is always an equal and opposite reaction.

If a body 'A' exerts force $\vec{F}$ on another body B, then B exerts a reaction force $\vec{R} = -\vec{F}$ on A. The two forces are acting along the same line.

Note :-



Action and reaction force acts on different bodies hence they never cancel each other.

* Newton's third law gives the nature of the force (i.e. forces always occur in pairs).