

Logic gate

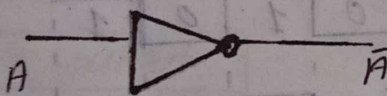
The electronics component which has ability to make some disk is turn is logic gate.

→ logic gate are 3 types :-

- i) Fundamental logic gate → NOT, OR, AND
- ii) Universal logic gate → NOR, NAND
- iii) Exclusive logic gate → EX-OR, EX-NOR

NOT GATE :- [IC - 7404]

Symbol :-



Input	Output
0	1
1	0

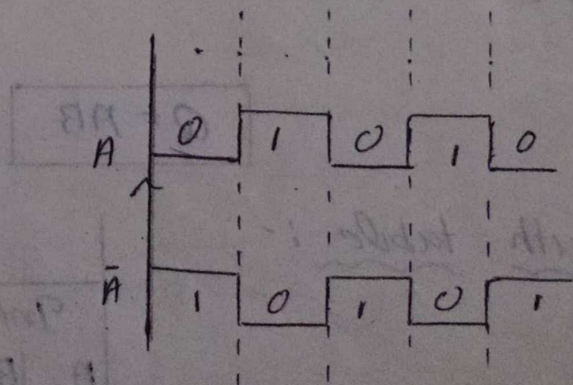
→ NOT gate have only one input & one output

Boolean expression :-

$$Q = \bar{A}$$

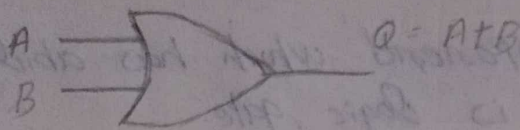
Truth table :-

Input (A)	Output (Q = $\bar{A}$)
0	1
1	0



OR GATE :- [IC - 7432]

Logic symbol :-



Boolean expression :-

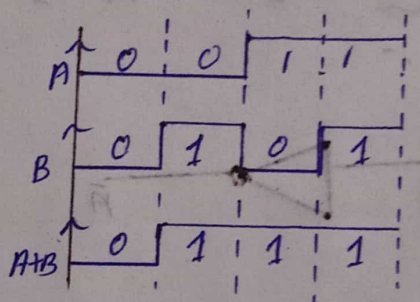
$$Q = A + B$$

It have 2 input and 1 output

Truth table

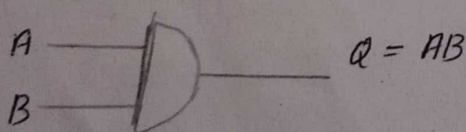
Input		output
A	B	(Q)
0	0	0
0	1	1
1	0	1
1	1	1

Timing diagram :-



AND GATE :- [IC - 7408]

Logic symbol :-



$$Q = AB$$

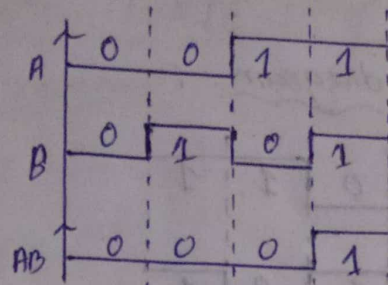
Boolean expression :-

$$Q = AB$$

Truth table :-

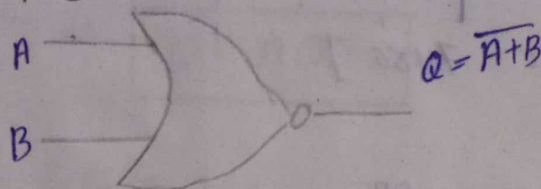
Input		output
A	B	(Q)
0	0	0
0	1	0
1	0	0
1	1	1

Timing diagram :-



NOR GATE :- [IC - 7402]

Logic symbol :-



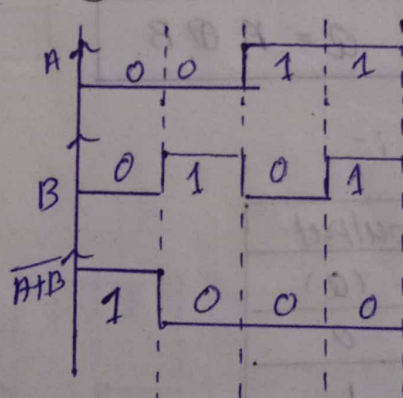
Boolean Expression :-

$$Q = \overline{A+B}$$

Truth table

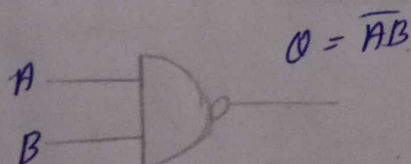
Input		output
A	B	(Q)
0	0	1
0	1	0
1	0	0
1	1	0

Timing diagram :-



NAND GATE :- [IC - 7400]

Logic symbol :-



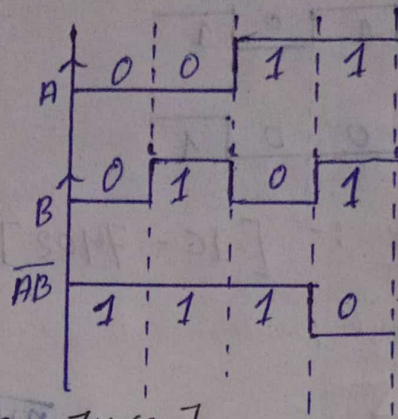
Boolean Expression :-

$$Q = \overline{AB}$$

Truth table :-

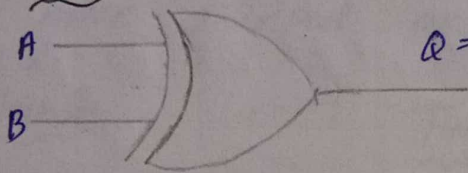
Input		output
A	B	(Q)
0	0	1
0	1	1
1	0	1
1	1	0

Timing diagram :-



EX-OR GATE :- [IC - 7486]

Logic symbol :



$$Q = A \oplus B$$

$$(\bar{A}B + A\bar{B})$$

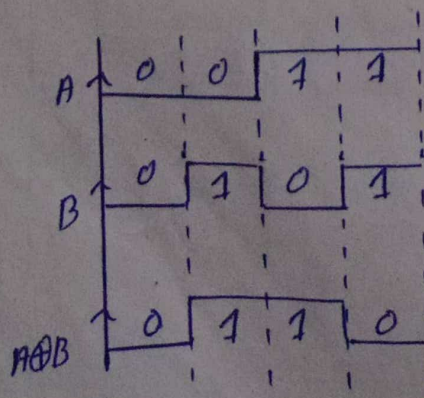
Boolean Expression :-

$$Q = A \oplus B$$

Truth table :-

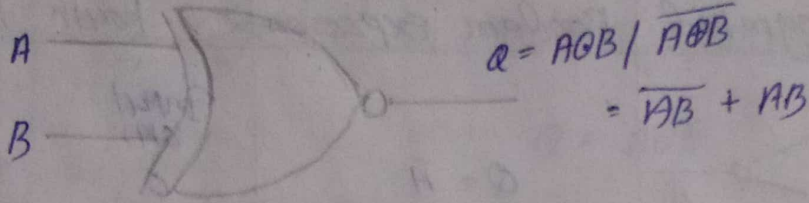
Input		output
A	B	(Q)
0	0	0
0	1	1
1	0	1
1	1	0

Timing diagram :-



EX - NOR GATE :- [IC - 74266]

Logic symbol :-



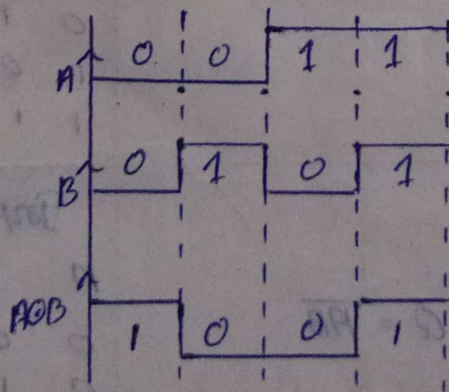
Boolean Expression :-

$$Q = A \oplus B$$

Truth table :-

Input		output
A	B	(Q)
0	0	1
0	1	0
1	0	0
1	1	1

Timing diagram :-



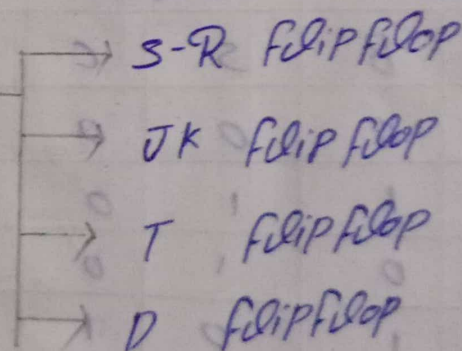
Combinational

- It is only depend upon the present input.
- It does not required feedback
- Ex :- Adder, subtractor
Encoder, decoder
multiplexers, demultiplexer

Sequential circuit :

- It is depend upon present input & past output.
- It Required feedback.

ex :- i) Flipflop



ii) counter

→ sequential
→ A sequential

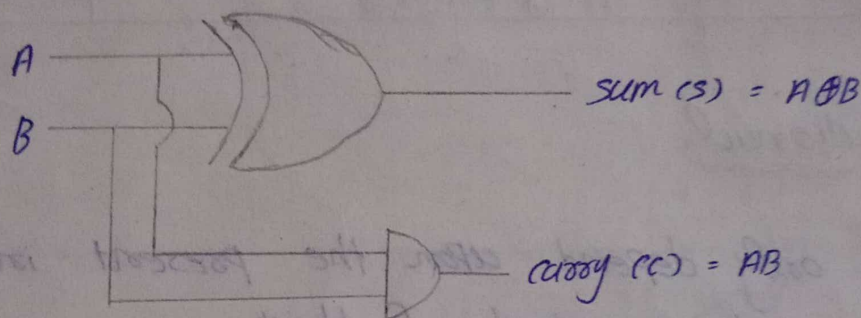
→ up counter
→ down counter

→ up counter
→ down counter

1-half - Adder :-

Half adder is a circuit in which two binary bit's are added and the resultant will be a two bit binary number. One bit in term's as some bit and other one is carry bit.

Logic symbol :-



No. of Input = 2 (A & B)

No. of output = 2 (sum (s),
carry (c))

Truth table :

Input		output	
A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

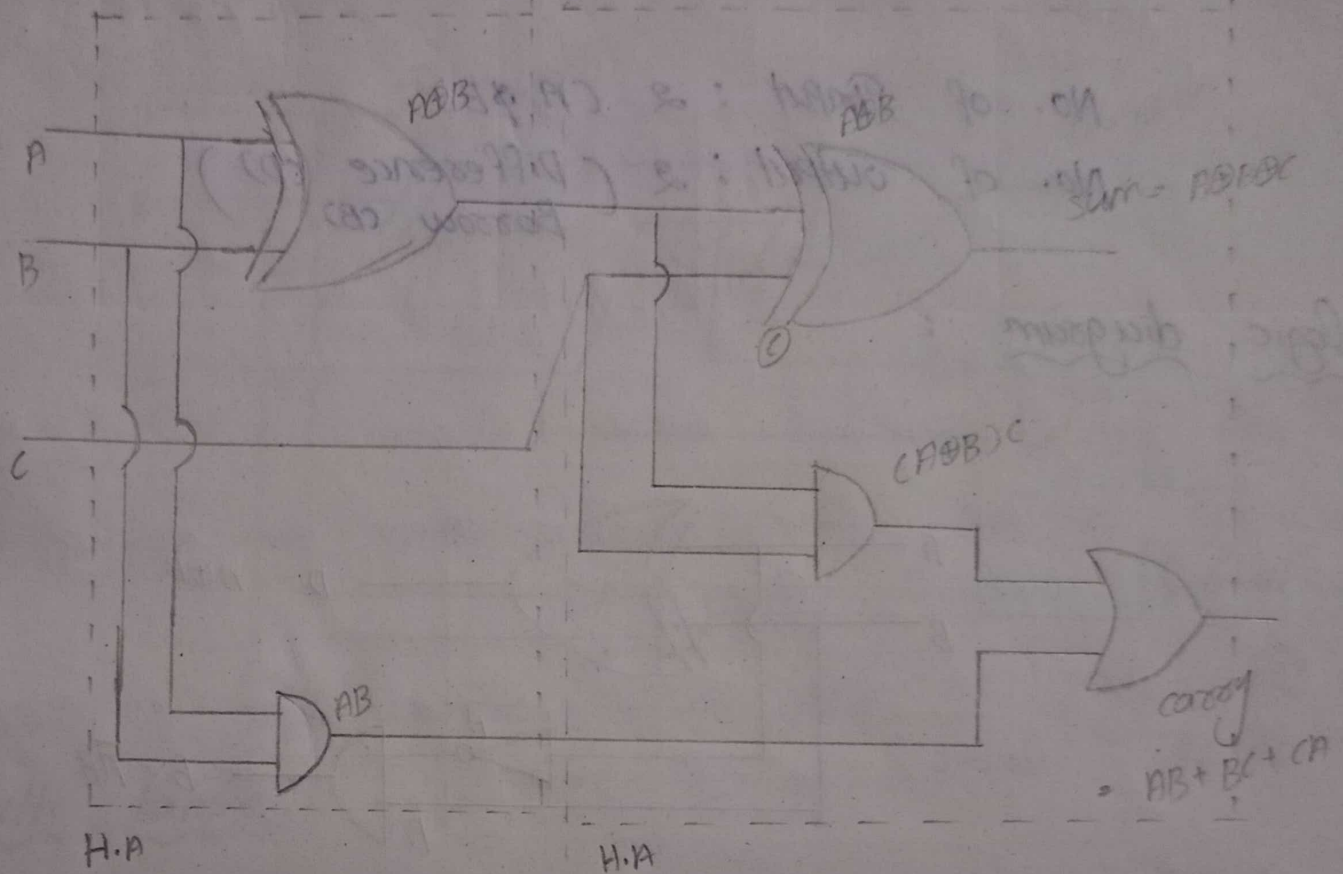
Full adder :

Full adder circuit is a circuit in which three binary bit's are added and resultant will be a two bit binary number.

No. of Input - 3 (A, B, C)

No. of output - 2 (sum, carry)

Logic diagram :



Truth table :

Input			output	
A	B	C	sum	carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

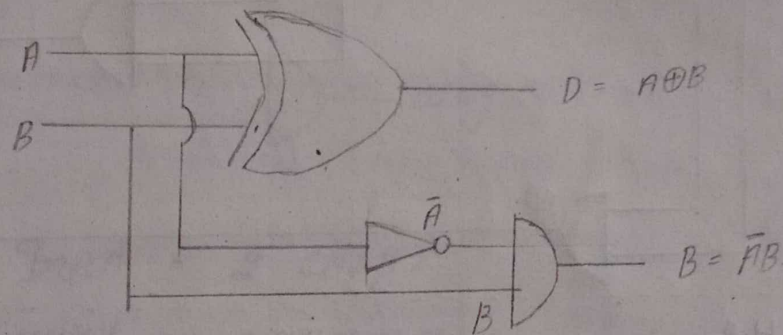
Half-subtractor :-

It is the combinational circuit in which we are finding the difference between 2 binary bit's and the resultant will be 2 bit.

No. of Input : 2 (A & B)

No. of output : 2 (Difference (D))
Borrow (B)

Logic diagram :



Truth table :-

Input		output	
A	B	D	B
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Full-subtractor :

It is a combinational circuit in which we are going to find the difference between three Binary bit than the resultant will be two bit.

No. of Input : 3 ($A \neq B \neq C$)
No. of output : 2 (Difference CD)
(Borrow CB)

Logic diagram :-

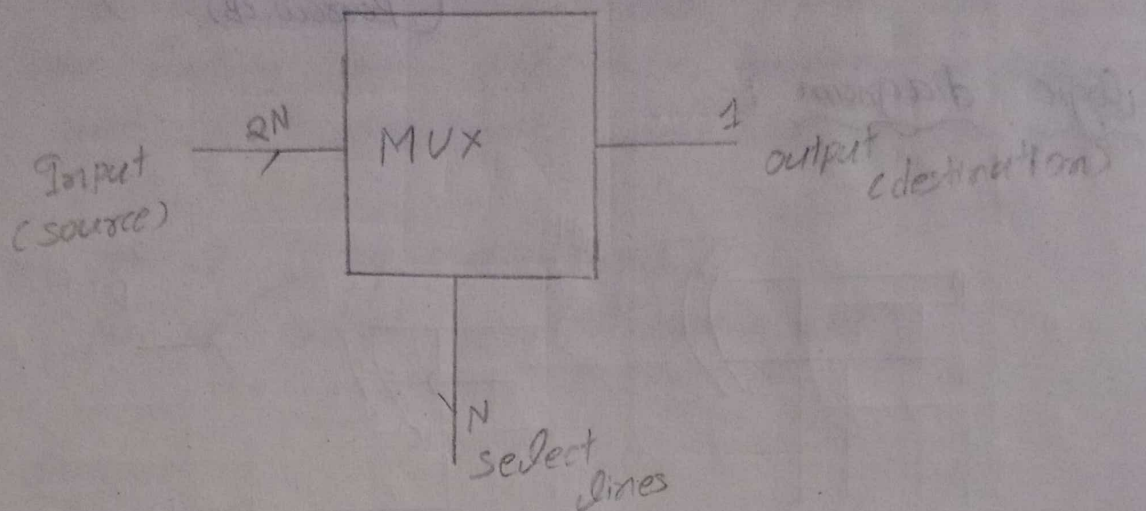
Tooth table:

Input			output	
A	B	C	A	B
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Multi Polycarbonate :- (Mux)

A multiplexer is a device that allows digital information from several sources to mean to be single line output for transmission.

A mux is a digital switch that has multiple inputs (source) and a single output (destination)



- It has several input lines
- Some select lines less than the no. of input lines
- single output line.
- If there are data input lines and M select lines, then $(2^M = n)$

MUX Types :-

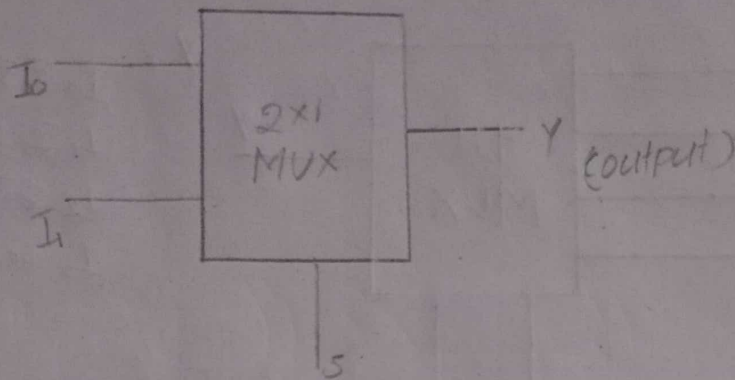
- 2 to 1 (1 select line)
- 4 to 1 (2 select line)
- 8 to 1 (3 select line)
- 16 to 1 (4 select line)

→ It is also called data selector

2:1 multiplexer :-

- 2:1 multiplexer is a combinational logic circuit that has only 2 input and one select line and one single output that is Y .

Logic symbol:



No. of input : 2 (I_0, I_1)

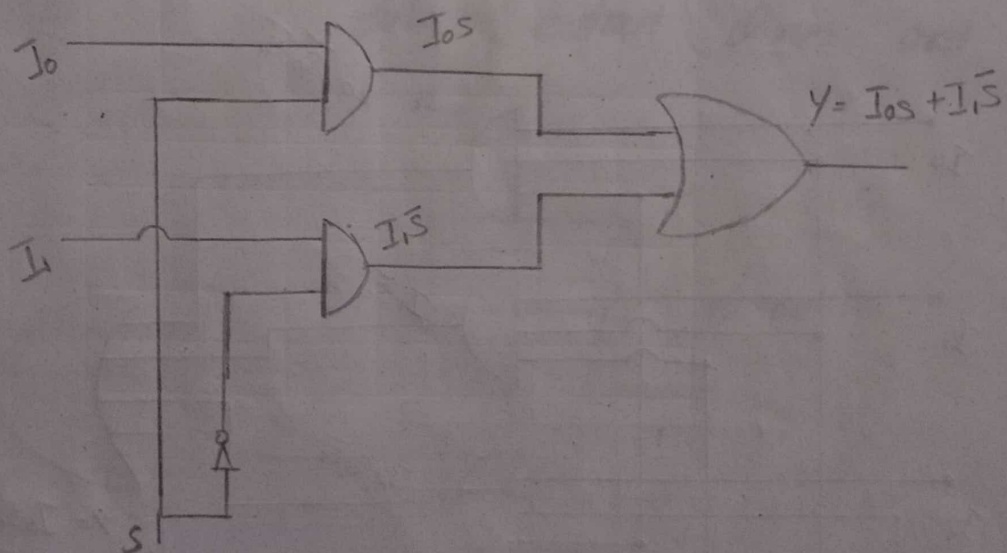
No. of select line : 1 (S)

No. of output line : 1 (Y)

Truth table :

S	Y
0	I_0
1	I_1

Logic diagram :-



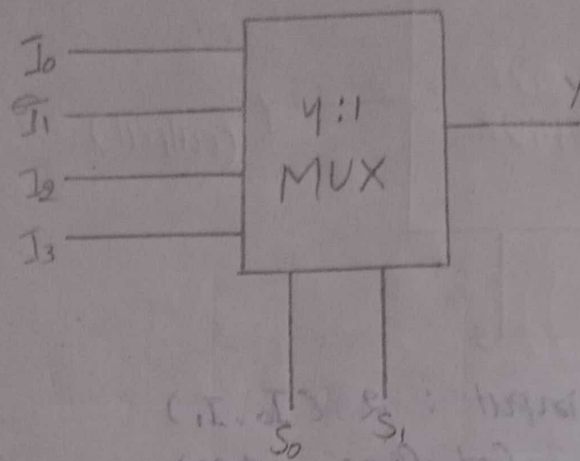
4:1 multiplexer :-

No. of Input :- 4 (I_0, I_1, I_2, I_3)

No. of output :- 1 (Y)

No. of select input :- 2 (S_0, S_1)

Logic symbol :-

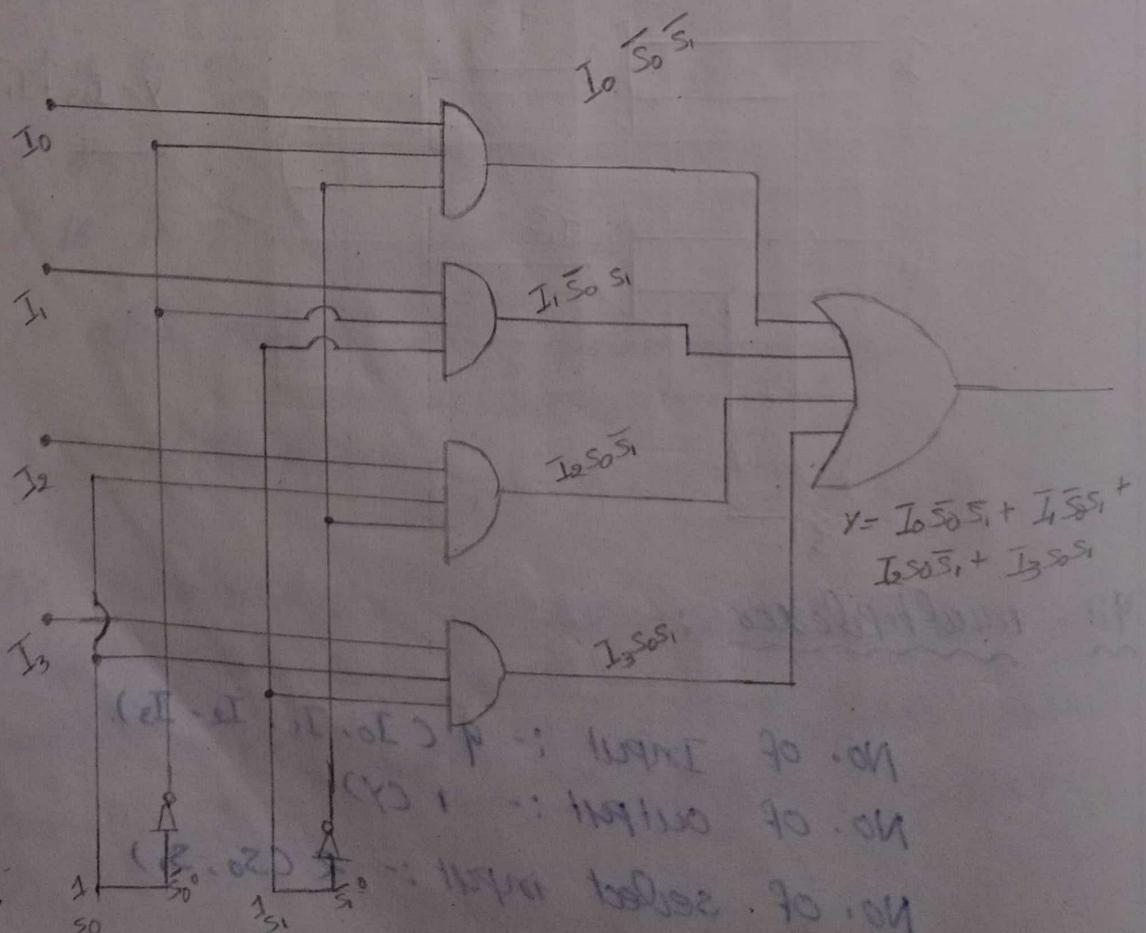


$$Y = I_0 \bar{S}_0 \bar{S}_1 + I_1 \bar{S}_0 S_1 + I_2 S_0 \bar{S}_1 + I_3 S_0 S_1$$

Truth table :-

Input		output
S_0	S_1	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

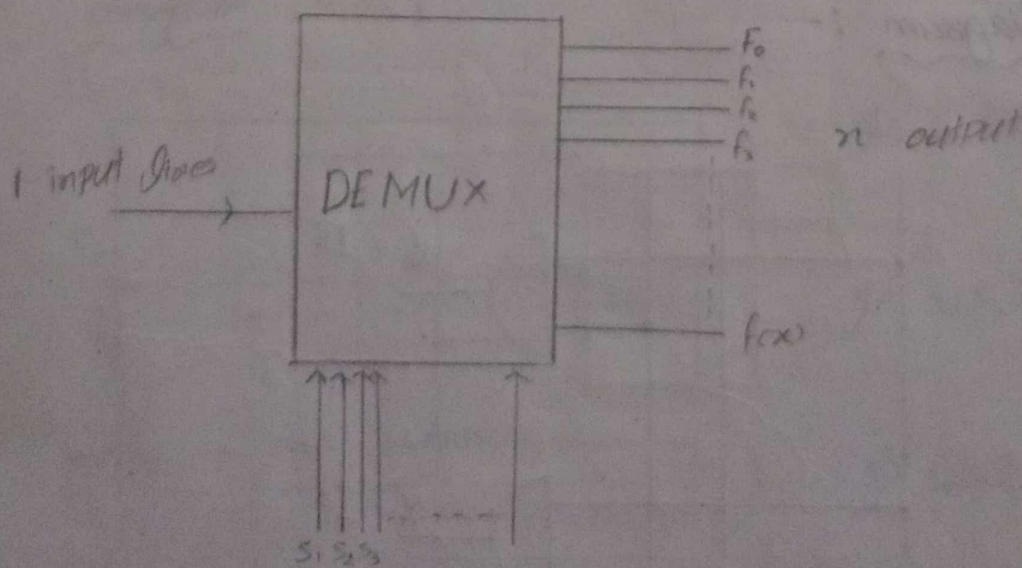
Logic Diagram :-



De-multiplexer :- (De-Mux)

- It takes data one line and distribute them when given number in output's line.
- From this reason the demultiplexer is also known as a data distribute device.
- single data input lines
- Some select line (less than the no. of output lines)
- several output lines.

where are a n data output lines and m select lines, ($2^m = n$)



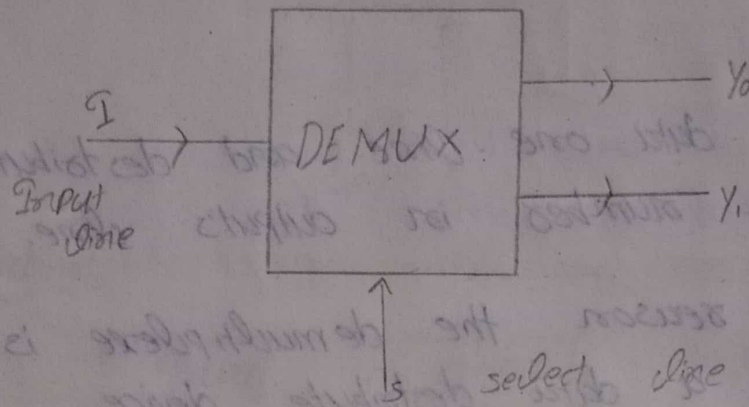
(m select line)

1:2 DE-MUX :-

No. of input :- 1 (I)

No. of output :- 2 (Y_0, Y_1)

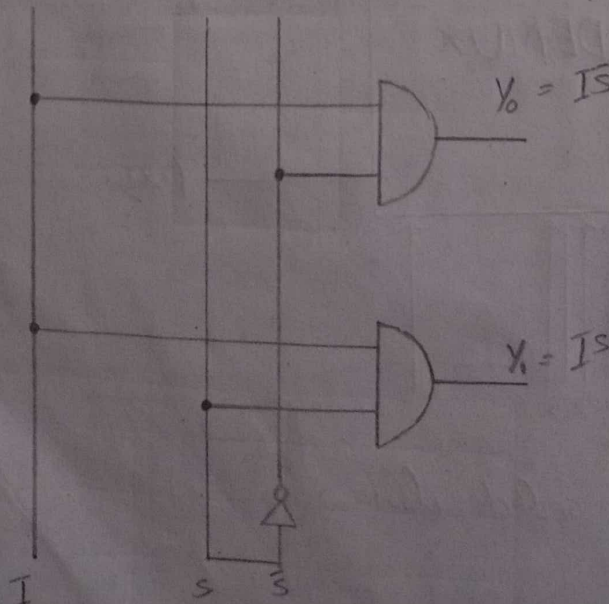
Basic symbol :-



Truth table :-

select line (s)	Input data (I)	Y_0	Y_1
0	I	1	0
1	I	0	1

Logic diagram :-



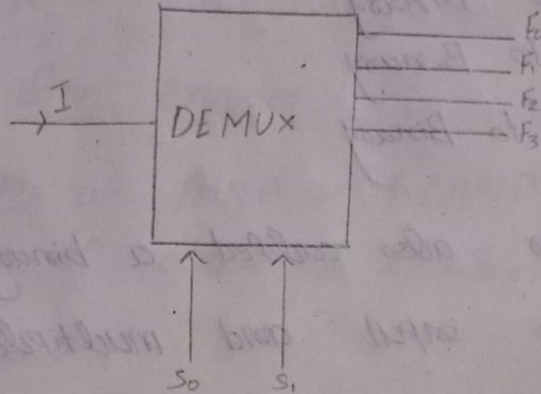
1:4 DEMUX :-

No. of output : 4 (F_0, F_1, F_2, F_3)

No. of input : 1 (I)

No. of select input : 2 (S_0, S_1)

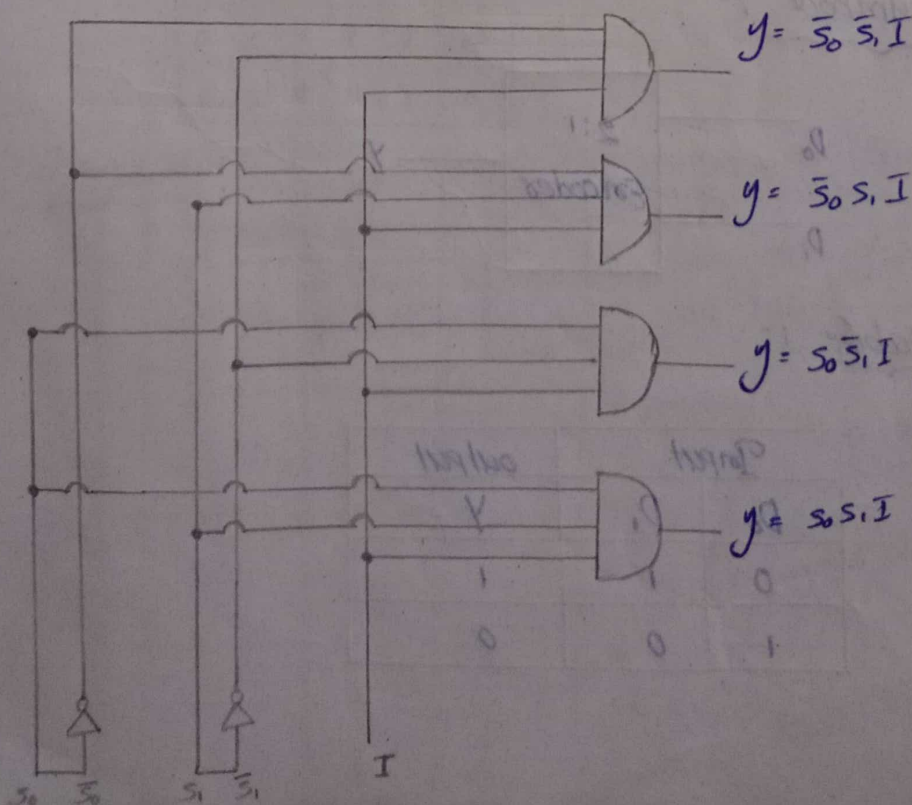
Basic symbol :



Truth table :-

Input			output			
E	S_0	S_1	F_0	F_1	F_2	F_3
0	x	x	0	0	0	0
1	0	0	I	0	0	0
1	0	1	0	I	0	0
1	1	0	0	0	I	0
1	1	1	0	0	0	I

Logic diagram :-



Encoder :-

A Encoder is a combinational circuit that convert's information from 1 code to another code.

Ex:- Octal to Binary
Decimal to Binary
Hexadecimal to Binary

- A Digital Encoder also called a binary Encoder.
- It has multiple input and multiple output device.
- Encoder in convert's 2^n line of input to n lines of output.

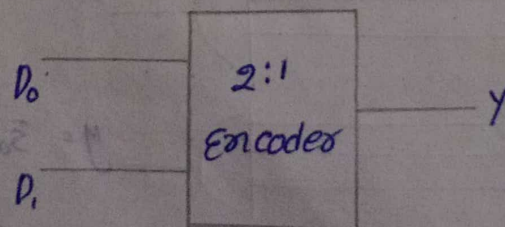
Ex:- 2:1, 4:2, 8:3, 16:4

2:1 Line Encoder :-

No. of Input : 2 (D_0, D_1)

No. of output : 1 (Y)

Logic symbol :-



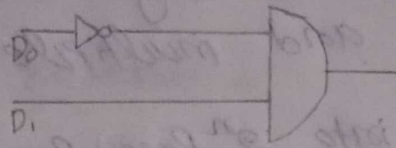
Truth table :-

Input		output
D_0	D_1	Y
0	1	1
1	0	0

Boudewor Expression :-

$$Y = \bar{D}_0 D_1$$

Logic diagram :-

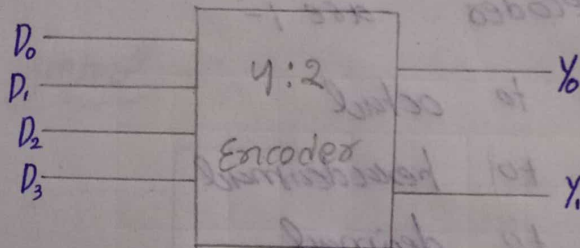


4:2 Line Encoder :-

No. of Input :- 4 (D_0, D_1, D_2, D_3)

No. of output :- 2 (Y_0, Y_1)

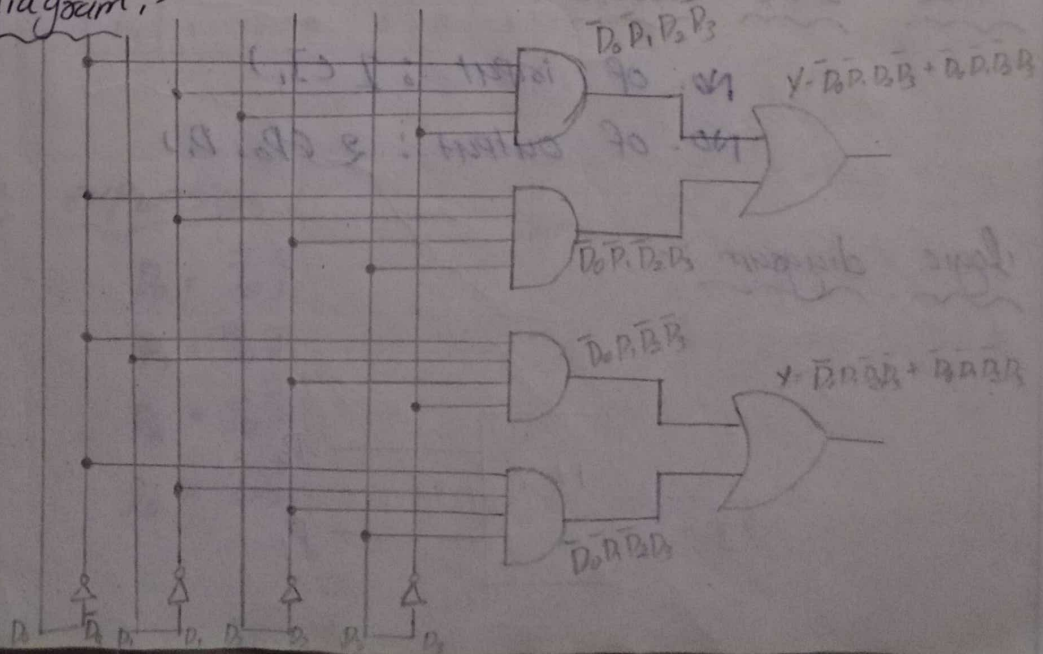
Logic symbol :-



Truth table :-

Input				Output	
D_0	D_1	D_2	D_3	Y_0	Y_1
1	0	0	0	0	0
0	1	0	0	0	1
0	0	1	0	1	0
0	0	0	1	1	1

Logic diagram :-

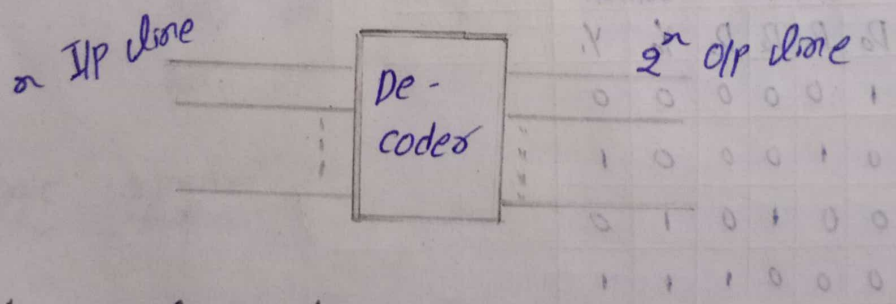


Decoder :-

- Decoder is a combinational logic circuit that have multiple input and multiple output.
- It decodes n input into 2^n possible output.
- The binary information is put the form of n input lines. The output line defined the 2^n bit code for the binary information.
- Application of decoder are :-

- Binary to octal
- Binary to hexadecimal
- Binary to decimal

Logic symbol :-

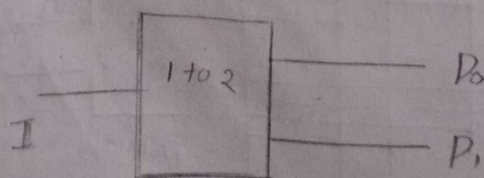


1 to 2 line decoder :-

No. of input : 1 (I_1)

No. of output : 2 (P_0, P_1)

Logic diagram :-



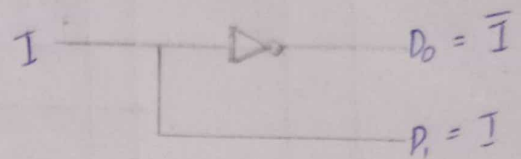
Truth table :-

I	D ₀	D ₁
0	1	0
1	0	1

Boolean Expression :-

$$D_0 = \bar{I}$$

$$D_1 = I$$

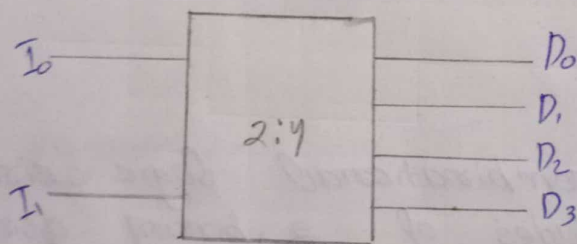


2 to 4 line decoder :-

No. of Input :- 2 (I₀, I₁)

No. of output :- 4 (D₀, D₁, D₂, D₃)

Logic symbol :-



Truth table :-

I/p		O/p			
I ₀	I ₁	D ₀	D ₁	D ₂	D ₃
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

Boolean expression :-

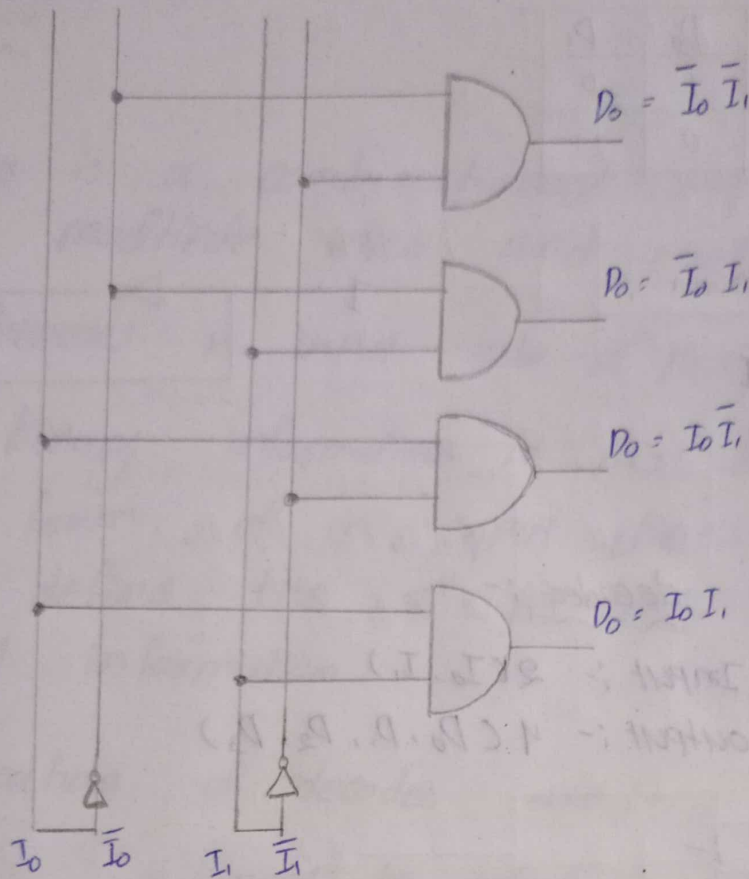
$$D_0 = \bar{I}_0 \bar{I}_1$$

$$D_1 = \bar{I}_0 I_1$$

$$D_2 = I_0 \bar{I}_1$$

$$D_3 = I_0 I_1$$

Logic diagram :-



Comparator :-

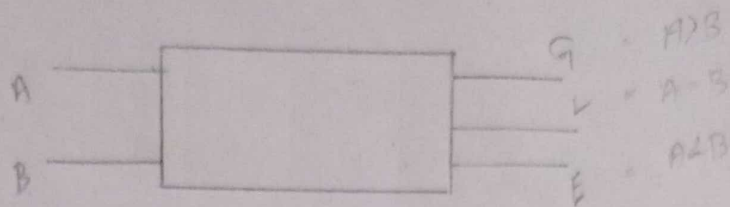
- Comparator is a combinational logic circuit that compares the magnitudes of 2 binary quantities (Numbers). To determine which 1 Number has less, =, greater magnitude.
- In other word a comparator determine the relation on 2 binary (Number)
- There are many type's of comparator
 - 1-bit magnitude comparator
 - 2-bit magnitude comparator
 - 3-bit magnitude comparator

1 bit magnitude comparator :-

No. of Input : 2 (A, B)

No. of output : 3 (G, L, E)

Basic symbol :-



Truth table :-

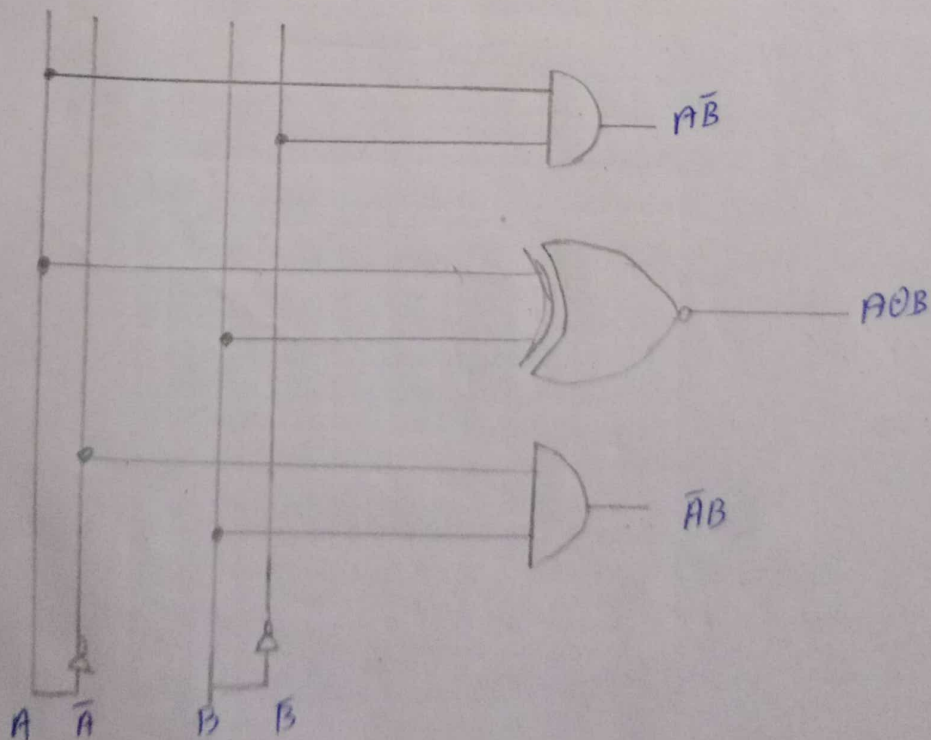
A	B	G $A \cdot B$	E $A - B$	L $A \oplus B$
0	0	0	1	0
0	1	0	0	1
1	0	1	0	0
1	1	0	1	0

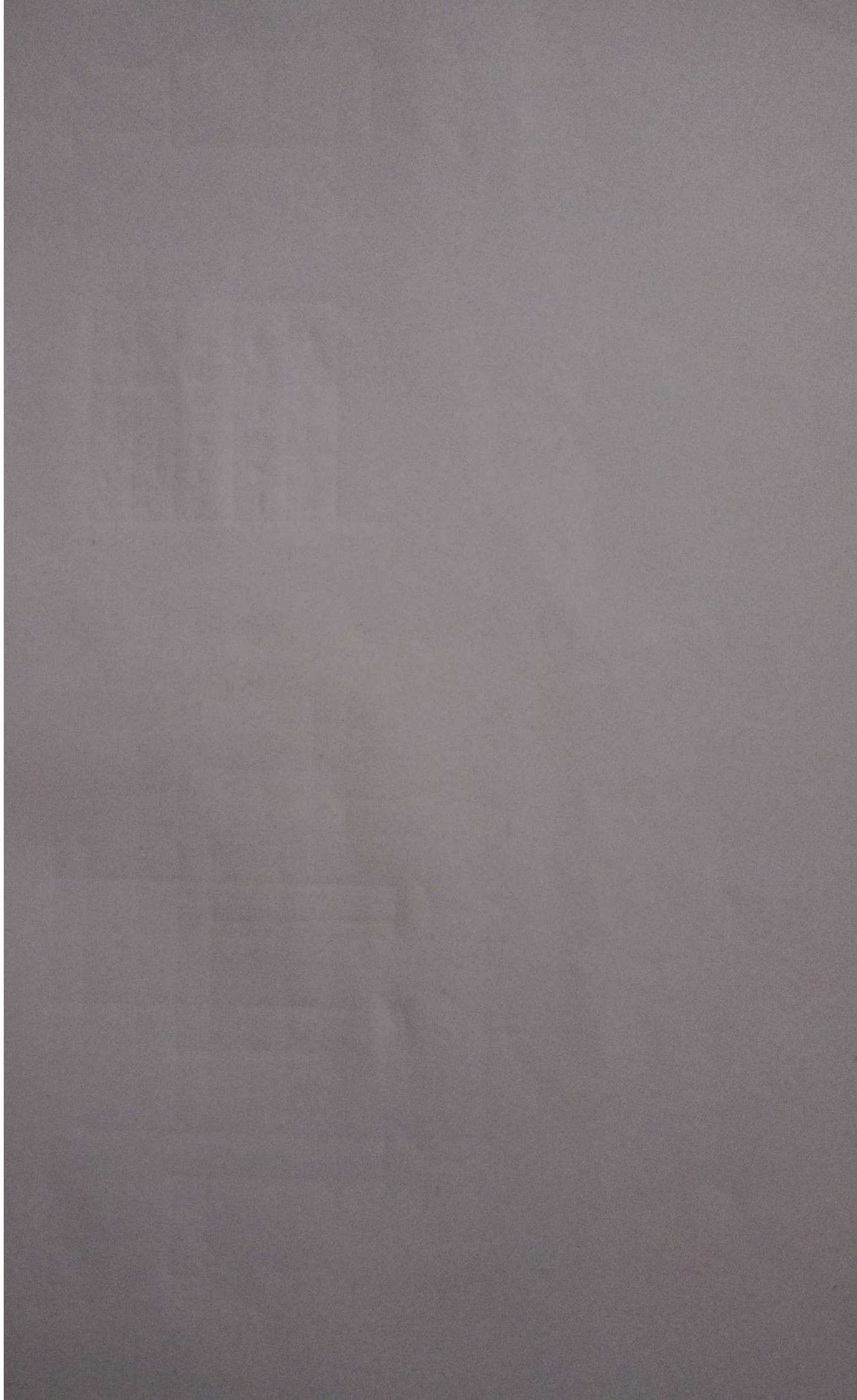
Boolean expression :-

$$G = A \cdot B, \quad E = \bar{A} \cdot \bar{B} + A \cdot B, \quad L = \bar{A} B$$

$A \odot B$

Logic diagram :-





Basic of Digital Electronics :-

What is Digital Electronics ?

- Digital Electronics is the study of electronic circuits that are used to process and control digital signal.

Number system :-

Introduction :-

- It is a system for representing numeric values or quantities using different symbols (digits)
- A Number system is a code that use symbols to refer to a number of items.
- The term digital refers to a process that is achieved by using discrete unit.
- In Number system there are different types of symbol has an absolute value and also has place value.

Radix / Base :-

- The radix or Base of a Number system is defined as the number of different digits which can occur in each position in the Number system.

Radix point :-

- The generalized form of a Decimal point is known as radix point. In any positional number system the radix point divides the integer and fractional part.

$$N_x = [\text{Integer part} \cdot \text{Fractional part}]$$

↑
Radix point

Number system :-

→ In general a number in a system having base or radix 'x' can be written as.

$$a_n a_{n-1} a_{n-2} \dots a_1 a_0 . a_{-1} a_{-2} \dots a_{-m}$$

This will be interpreted as :

$$y = a_n \times x^n + a_{n-1} \times x^{n-1} + a_{n-2} \times x^{n-2} + \dots + a_0 \times x^0 + a_{-1} \times x^{-1} + a_{-2} \times x^{-2} + \dots + a_{-m} \times x^{-m}$$

where; y = value of the entire number

a_n = The values of the nth digit

x = radix

Types of Number system :-

There are generally four types of number systems they are;

- i) Decimal Number system
- ii) Binary Number system
- iii) octal Number system
- iv) Hexadecimal Number system

i) Decimal Number system :-

→ The decimal number system contains ten unique symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

→ In decimal number system 10 symbols are involved so the base or radix is 10.

→ The decimal Number system is a radix-10 Number system and therefore has 10 different digits or symbols. It is a positional weighted Number system.

→ All higher Number of ex (a) are represented in terms of these ten digits only.

Example : $(42.73)_{10}$, $(824.36)_{10}$

ii) Binary Number system :-

→ The binary Number system is a positional weighted system

→ The base or radix of this Number system is 2.

→ It has two independent symbols that are 0 and 1.

→ A binary digit is called a bit.

Example : $(10101.110)_2$

iii) Octal Number system :-

→ It is also a positional weighted system.

→ Its base or radix is 8. so it has 8 independent symbols are 0, 1, 2, 3, 4, 5, 6, 7.

→ Its base $8 = 2^3$, every 3-bit group of binary can be represented by an octal digit.

Example : $(82.64)_8$, $(72.32)_8$

iv) Hexadecimal Number system :-

→ The Hexadecimal Number system is a positional weighted system.

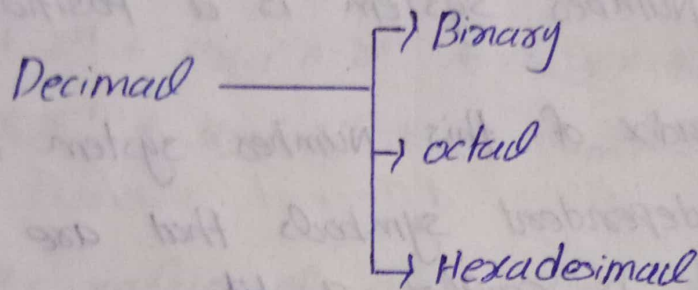
→ The base or radix of this Number system is 16 such as 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

↓ ↓ ↓ ↓ ↓ ↓
10 11 12 13 14 15

→ The base $16 = 2^4$, every 4-bit group of binary can be represented by a hexadecimal digit.

Examples :- $(AC.F3)_{16}$, $(2B.CD)_{16}$

CONVERSION FROM ONE NUMBER SYSTEM TO ANOTHER :-



[Rule : Division & multiplication]

Convert Decimal to other Number system :-

Decimal to Binary conversion :-

- Divide the given Decimal Number, successively by 2 and read the integer part remainder upwards to get equivalent binary Number.
- Multiply the fraction part by 2. keep the integer in the product as it and multiply the new fraction in the product by 2. The process is continued and the integers are read in the products from top to bottom.

Example

Q/ convert $(19.35)_{10}$ into binary.

$$\begin{array}{r} 2 \overline{) 19} \quad \text{LSB} \\ 2 \overline{) 9} \quad -1 \\ 2 \overline{) 4} \quad -1 \\ 2 \overline{) 2} \quad -0 \\ \quad \quad 1 \quad \quad \quad \uparrow \\ \text{MSB} \end{array} \quad \begin{array}{l} \text{LSB} \rightarrow \text{(Least significant Bit)} \\ \text{MSB} \rightarrow \text{(Most significant Bit)} \end{array}$$
$$(19.35)_{10} = (1011.0101)_2$$

$$0.35 \times 2 = 0.70$$

$$0.70 \times 2 = 1.40$$

$$0.40 \times 2 = 0.80$$

$$0.80 \times 2 = 1.60$$

Decimal to octal conversion :-

→ To convert the given decimal integer number to octal, successively divide the number by 8 till the quotient is 0.

→ To convert the given decimal fractions to octal, successively multiply the decimal fraction and the subsequent decimal fractions by 8 till the product is 0 or the required accuracy is obtained.

Example :-

Q/ convert $(378.93)_{10}$ into octal.

$$\begin{array}{r} 8 \overline{) 378} \\ 8 \overline{) 47} \quad -2 \\ 8 \overline{) 5} \quad -7 \\ \quad \quad 0 \quad -5 \end{array} \quad \uparrow$$

$$0.93 \times 8 = 7.44$$

$$0.44 \times 8 = 3.52$$

$$0.52 \times 8 = 4.16$$

$$0.16 \times 8 = 1.28$$

$$(378.93)_{10} = (572.7341)_8$$

Decimal to Hexadecimal conversion :-

→ The decimal to hexadecimal conversion is same as octal.

Example :-

convert $(2598.675)_{10}$ into Hexadecimal.

$$16 \overline{) 2598}$$

$$16 \overline{) 162} - 6$$

$$16 \overline{) 10} - 2$$

$$0 \rightarrow 10$$

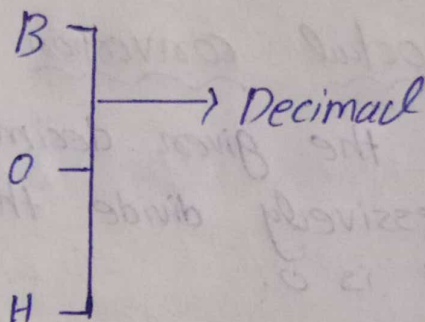
$$0.675 \times 16 = 10.8 \quad A$$

$$0.800 \times 16 = 12.8 \quad C$$

$$0.800 \times 16 = 12.8 \quad C$$

$$(2598.675)_{10} = (CA26.ACC)_{16}$$

Convert other number system Decimal :-



Binary to Decimal conversion :-

→ In this method, each binary of the number is multiplied by its positional weight and the product terms are added to obtain decimal Number.

Example :

i) convert $(10101)_2$ to Decimal

$$1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$\Rightarrow 16 + 0 + 4 + 0 + 1$$

$$\Rightarrow (21)_{10}$$

$$(10101)_2 = (21)_{10}$$

i) convert $(111.101)_2$ to Decimal

$$1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$$
$$\Rightarrow 4 + 2 + 1 + 0.5 + 0 + 0.125$$

$$\Rightarrow (7.625)_{10}$$

$$(111.101)_2 = (7.625)_{10}$$

Octal to Decimal conversion :-

→ For conversion octal to Decimal Numbers, multiply each digit in the octal Number by the weight of its position and add all the product terms.

Example :

i) convert $(4057.06)_8$ to Decimal

$$4 \times 8^3 + 0 \times 8^2 + 5 \times 8^1 + 7 \times 8^0 + 0 \times 8^{-1} + 6 \times 8^{-2}$$

$$\Rightarrow 2048 + 0 + 40 + 7 + 0 + 0.0937$$

$$\Rightarrow (2095.0937)_{10}$$

Hexadecimal to Decimal conversion :

→ For conversion of Hexadecimal to decimal multiply each digit in the hexadecimal Number by its position weight and also those product terms.

Example :

Convert $(B9F.AE)_{16}$ to Decimal.

$$B \times 16^2 + 9 \times 16^1 + F \times 16^0 + A \times 16^{-1} + E \times 16^{-2}$$

$$\Rightarrow 2816 + 144 + 16 + 0.628 + 0.058$$

$$\Rightarrow 2976.683$$

$$(B9F.AE)_{16} = (2976.683)_{10}$$

Binary	octal	Hexadecimal	decimal
0000	0	0	0
0001	1	1	1
0010	2	2	2
0011	3	3	3
0100	4	4	4
0101	5	5	5
0110	6	6	6
0111	7	7	7
1000	8	8	8
1001	9	9	9
1010	10	A	10
1011	11	B	11
1100	12	C	12
1101	13	D	13
1110	14	E	14
1111	15	F	15

Binary addition :-

The Binary Addition rules are

$$0+0=0$$

$$0+1=1$$

$$1+0=1$$

$$1+1=10 \text{ [ie 0 with a carry 1]}$$

Binary subtraction :-

The Binary subtraction rules are,

$$0-0=0$$

$$0-1=1$$

(with a borrow of 1)

$$1-0=1$$

$$1-1=0$$

Binary multiplication :-

The Binary multiplication rules are,

$$0 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 0 = 0$$

$$1 \times 1 = 1$$

Q/multiplication $(1101)_2$ by $(110)_2$

$$\begin{array}{r} 1101 \\ \times 110 \\ \hline 0000 \\ 1101 \\ 1101 \\ \hline 100110 \end{array}$$

1's complement Representation :-

The 1's complement of a Binary Number is obtain by changing each

0 $\rightarrow$ 1

1 $\rightarrow$ 0

Q Find $(1100)_2$ 1's complement

1 1 0 0

$\downarrow \downarrow \downarrow \downarrow$

0 0 1 1

$\leftarrow$ 1's complement

Addition by 1's complement method :-

7 Add +4 and +4 by 1's complement method

$$\begin{array}{r} 4 \rightarrow 0100 \\ + 4 \rightarrow 1001 \\ \hline 13 \quad 1101 \end{array}$$

7 Add +3 and -8 by 1's complement method

$$\begin{array}{r} +3 \rightarrow 0011 \rightarrow 0011 \\ + -8 \rightarrow 1000 \rightarrow 0111 \\ \hline 1010 \end{array}$$

1's complement $\rightarrow$ 0101

i.e. 0101 = (5)

So the sum of +3 and -8 is -5.

[As -ve number, so 1's complement of 8 (1000 = 0111)]

→ Add -6 and 2 by using 2's complement

$$\begin{array}{r} -6 \rightarrow 110 \rightarrow 001 \\ + 2 \rightarrow 010 \rightarrow 010 \\ \hline 011 \\ \text{C, } 100 \rightarrow 4 \end{array}$$

Subtraction by using complement method :-

→ Subtraction 7 from 21 by 2's complement

$$21 \rightarrow 10101 \rightarrow 10101$$

$$7 \rightarrow 00111 \rightarrow 11000$$

$$\begin{array}{r} 10101 \\ \hline \end{array}$$

$$\text{C, } +1$$

$$\begin{array}{r} 01110 \rightarrow (14)_2 \end{array}$$

→ Subtraction $(10000)_2$ from $(11010)_2$ by 1's complement

$$11010 \rightarrow 11010$$

$$10000 \rightarrow 01111$$

$$\begin{array}{r} 11010 \\ \hline \end{array}$$

$$\text{C, } +1$$

$$\begin{array}{r} 1010 \rightarrow 10 \end{array}$$

2's complement Representation :-

The 2's complement of a binary number is a binary number which is obtained by adding 1 to the 1's complement of a number.

$$2's \text{ complement} = 1's \text{ complement} + 1$$

Example :-

Find $(101101)_2$ 2's complement

$$\begin{array}{r} 010010 \\ \underline{ 1} \end{array}$$

Result $\rightarrow (010011)_2$

Law's of Boolean Algebra :-

Complementation	OR Laws	AND Laws
Law-1: $\bar{0} = 1$	$A+0 = A$ (Null Law)	$A \cdot 0 = 0$ (Null Law)
Law-2: $\bar{1} = 0$	$A+1 = 1$ (Identify Law)	$A \cdot 1 = A$ (Identify Law)
Law-3: If $A=0$ then $\bar{A}=1$	$A+A = A$	$A \cdot A = A$
Law-4: If $A=1$ then $\bar{A}=0$	$A+\bar{A} = 1$	$A \cdot \bar{A} = 0$
Law-5: $\bar{\bar{A}} = A$		

DeMorgan's Theorem :-

1st Law states that the complement of a sum of variable is equal to the product of these individual complement.

$$\boxed{\overline{A+B} = \bar{A} \cdot \bar{B}}$$

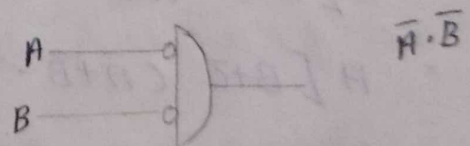
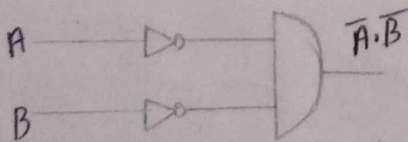
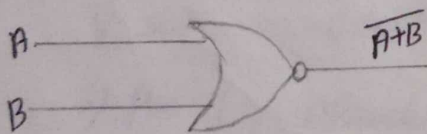
A	B	A+B	$\overline{A+B}$	$\overline{A} + \overline{B}$	$\overline{A} \cdot \overline{B}$
0	0	0	1	1	1
0	1	1	0	1	0
1	0	1	0	0	0
1	1	1	0	0	0

2nd law state that the complement of a product of variable is equal to the sum of their individual complement.

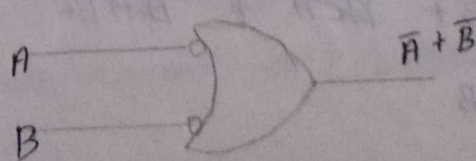
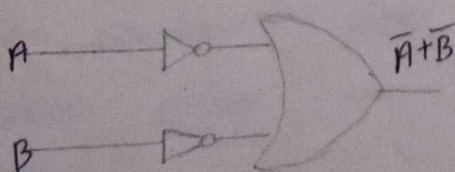
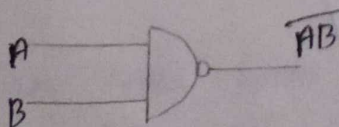
$$\overline{AB} = \overline{A} + \overline{B}$$

A	B	AB	$\overline{AB}$	$\overline{A}$	$\overline{B}$	$\overline{A} + \overline{B}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

Nor gate is equivalent to bubbled AND Gate:-



NAND gate is equivalent to bubbled OR gate:-



Reducing Boolean Expression :-

1)

$$\begin{aligned} & \overline{AB + (A+B)} \\ & (\overline{AB} + \overline{A+B}) \\ & (\overline{A} + \overline{B}) (\overline{A} \cdot \overline{B}) \end{aligned}$$

2)

$$\begin{aligned} Y &= \overline{(A+B)(C+D)} \\ &= \overline{(A+B)} + \overline{(C+D)} \\ &= (\overline{A} \cdot \overline{B}) + (\overline{C} + \overline{D}) \\ &= \overline{A}B + \overline{C} \cdot D \end{aligned}$$

3)

$$\begin{aligned} Y &= \overline{\overline{AB} + \overline{A} + AB} \\ &= \overline{\overline{AB}} + \overline{\overline{A}} + \overline{AB} \\ &= AB \cdot A \cdot \overline{A} + \overline{B} \\ &= AB \cdot \overline{A} + \overline{B} \\ &= \overline{A}AB \cdot AB\overline{B} \\ &= 0B \cdot A0 \\ &= 0 \end{aligned}$$

$$\begin{aligned} 6) &= \overline{(A+B\overline{C})} (\overline{A\overline{B}} + \overline{AB\overline{C}}) \\ &= (\overline{A} + \overline{B\overline{C}}) (\overline{A\overline{B}} + \overline{AB\overline{C}}) \\ &= (\overline{A} \cdot B\overline{C}) (\overline{A} + \overline{B} \cdot \overline{AB\overline{C}}) \\ &= \overline{A}B\overline{C} (\overline{A\overline{B}} + \overline{AB\overline{C}}) \\ &= \overline{A}B\overline{C} \overline{A\overline{B}} + \overline{A}B\overline{C} \overline{AB\overline{C}} \\ &= 0 \end{aligned}$$

4)

$$\begin{aligned} Y &= A [B + \overline{C} (\overline{AB} + \overline{AC})] \\ &= A [B + \overline{C} (\overline{AB} + \overline{AC})] \\ &= A [B + \overline{C} (\overline{A} + \overline{B} \cdot \overline{A} + \overline{C})] \\ &= A [B + \overline{C} (\overline{A} + \overline{AC} + \overline{B}\overline{A} + \overline{BC})] \\ &= A [B + (\overline{C}\overline{A}) + \overline{A}\overline{C}\overline{C} + \overline{A}\overline{B}\overline{C} + \overline{B}\overline{C}\overline{C}] \\ &= A [B + (\overline{C}\overline{A} + \overline{A}\overline{B}\overline{C})] \\ &= AB + \overline{A}\overline{C}\overline{A} + \overline{A}\overline{A}\overline{B}\overline{C} \\ &= AB \end{aligned}$$

$$\begin{aligned}
\rightarrow F &= (B + \overline{B}C) (CB + \overline{B}C) (CB + D) \\
&= B(CB + \overline{B}C) + \overline{B}(CB + \overline{B}C) (CB + D) \\
&= [B\overline{B}C + B\overline{B}C + BCB + B\overline{B}C] (CB + D) \\
&= (CB + \overline{B}C) (CB + D) \\
&= BB + BD + BCB + BCD \\
&= B + BD + BCB + BCD \\
&= B + BC + BD + BCD \\
&= B(1+C) + BD(1+C) \\
&= B + BD \\
&= B(1+D) \\
&= B
\end{aligned}$$

Boolean Expression and their Representation :-

There are different base of Representing a given function is 2 way in the following way.

Standard form ;

i) SOP :- (sum of Product)

ii) POS :- (Product of sum)

SOP :-

This form is also known as is also called Disjunctive normal form (DNF)

7 In this form the function is the sum of a number of product term where each product term contains all the variable of the function either in complemented or uncomplemented form.

→ A product term which contains all the variables of the function either in complemented or uncomplemented form is called a MINTERM (Σ_m)

Ex:- $F(A, B, C) = \bar{A}B + \bar{B}C$

$$\bar{A}B(C + \bar{C}) + (A + \bar{A})\bar{B}C$$

$$= \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}C + \bar{A}\bar{B}C$$

$$= 011, 010, 001, 101$$

$$3, 2, 1, 5$$

$$\Sigma_m = (1, 2, 3, 5)$$

Pos :- (Π_m)

This form is also called as conjunctive canonical form (ccf)

→ A sum term which contains all variables either complemented or uncomplemented form is called a MAXTERM.

Ex:- $F(A, B, C) = (\bar{A} + \bar{B})(B + C)$

$$(\bar{A} + \bar{B} + C \cdot \bar{C})(A \cdot \bar{A} + B + C)$$

$$= (\bar{A} + \bar{B} + C)(\bar{A} + \bar{B} + \bar{C})(A + B + C)(\bar{A} + B + C)$$

$$= 110, 111, 000, 100$$

$$= 6, 7, 0, 4$$

$$\Pi_m = (0, 4, 6, 7)$$

If the boolean expression are representing in terms of MINTERM and MAXTERM then it is form of canonical form.

i) MINTERM (Σm)

ii) MAXTERM (ΠM)

3- variable Expression :-

a	b	c	MINTERM (m)	MAXTERM (M)
0	0	0	$m_0 (\bar{a}\bar{b}\bar{c})$	$M_0 (a+b+c)$
0	0	1	$m_1 (\bar{a}\bar{b}c)$	$M_1 (a+b+\bar{c})$
0	1	0	$m_2 (\bar{a}b\bar{c})$	$M_2 (a+\bar{b}+c)$
0	1	1	$m_3 (\bar{a}bc)$	$M_3 (a+\bar{b}+\bar{c})$
1	0	0	$m_4 (a\bar{b}\bar{c})$	$M_4 (\bar{a}+b+c)$
1	0	1	$m_5 (a\bar{b}c)$	$M_5 (\bar{a}+b+\bar{c})$
1	1	0	$m_6 (ab\bar{c})$	$M_6 (\bar{a}+\bar{b}+c)$
1	1	1	$m_7 (abc)$	$M_7 (\bar{a}+\bar{b}+\bar{c})$

MINTERMS

→ Each individual form is SOP is called as MINTERM

→ In this form '1' is used for representing the MINTERM

MAXTERMS

→ Each individual form is POS is called as MAXTERM.

→ In this form '0' representing the MAXTERM

Conversion from standard form to conic form:

$$\begin{aligned}\Rightarrow f(A, B, C) &= ab + b\bar{c} \\&= \bar{a}b(c + \bar{c}) + (a + \bar{a})b\bar{c} \\&= \bar{a}bc + \bar{a}b\bar{c} + ab\bar{c} + a b\bar{c} \\&= 011, 010, 110, 010 \\&= 3, 2, 6, 2\end{aligned}$$

$$\Sigma_m = (2, 3, 6)$$

$$\begin{aligned}\Rightarrow f(x, y, z) &= x + \bar{y}z + xy\bar{z} \\&= x(y + \bar{y})(z + \bar{z}) + (x + \bar{x})\bar{y}z + xy\bar{z} \\&= xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + x\bar{y}z + \bar{x}\bar{y}z + xy\bar{z} \\&= 111, 110, 101, 100, 101, 001, 110 \\&= 7, 6, 5, 4, 5, 1, 6 \\&\Sigma_m = (1, 4, 5, 6, 7)\end{aligned}$$

$$\begin{aligned}\Rightarrow f(P, Q, R) &= P\bar{R} + QR + P\bar{Q}R \\&= P(Q + \bar{Q})\bar{R} + (P + \bar{P})QR + P\bar{Q}R \\&= P\bar{Q}\bar{R} + P\bar{Q}\bar{R} + PQR + \bar{P}QR + P\bar{Q}R \\&= 110, 100, 111, 011, 101 \\&= 6, 4, 7, 3, 5 \\&\Sigma_m = (3, 4, 5, 6, 7)\end{aligned}$$

$$\begin{aligned}
 \Rightarrow f(a, b, c) &= (a + b)(a + c) \\
 &= a + b(c \cdot \bar{c}) + a(b \cdot \bar{b})\bar{c} \\
 &= (a + b + c)(a + b + \bar{c})(a + b + c)(a + \bar{b} + \bar{c}) \\
 &= 100, 101, 001, 011 \\
 &= 4, 5, 1, 3 \\
 \pi_m &= (1, 3, 4, 5)
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow f(x, y, z) &= y(x + \bar{z}) \\
 &= (x \cdot \bar{x}) + y + (z \cdot \bar{z})(x + (y \cdot \bar{y}) + \bar{z}) \\
 &= (x + y + z)(x + y + \bar{z})(\bar{x} + y + z)(\bar{x} + y + \bar{z})(x + y + \bar{z}) \\
 &\quad (x + \bar{y} + \bar{z}) \\
 &= 000, 001, 100, 101, 001, 011 \\
 &= 0, 1, 4, 5, 1, 3 \\
 \pi_m &= (0, 1, 3, 4, 5)
 \end{aligned}$$

$$\Rightarrow \text{A}(\text{B} + \bar{\text{B}}) \text{C}$$

$$\begin{aligned}
 \Rightarrow f(A, B, C) &= A + B\bar{C} \\
 &= A(B + \bar{B})(C + \bar{C}) + (A + \bar{A})B\bar{C} \\
 &= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}B\bar{C} \\
 &= 111, 110, 101, 100, 110, 010 \\
 &= 7, 6, 5, 4, 6, 2 \\
 \sum_m &= (2, 4, 5, 6, 7)
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow f(x, y, z) &= xz + \bar{y}z \\
 &= x(y + \bar{y})z + (x + \bar{x})\bar{y}z \\
 &= xyz + x\bar{y}z + x\bar{y}z + \bar{x}\bar{y}z \\
 &= 111, 101, 101, 001 \\
 &= 7, 5, 5, 1 \\
 \sum_m &= (1, 5, 7)
 \end{aligned}$$

$$\begin{aligned} \Rightarrow F(P, Q, R, S) &= P + QS + RS + \overline{Q}\overline{R}S \\ &= P(Q + \overline{Q})(R + \overline{R})(S + \overline{S}) + (P + \overline{P})Q(R + \overline{R})S + (P + \overline{P})(Q + \overline{Q}) \\ &\quad (R + \overline{R})S + (P + \overline{P})Q\overline{R}S \end{aligned}$$

$$\begin{aligned} &= PQR S + PQR \overline{S} + P\overline{Q}R S + P\overline{Q}R \overline{S} + P\overline{Q}\overline{R} S + P\overline{Q}\overline{R} \overline{S} + P\overline{Q}\overline{R} S \\ &\quad + P\overline{Q}\overline{R} \overline{S} + PQR S + PQR \overline{S} + P\overline{Q}R S + P\overline{Q}R \overline{S} + P\overline{Q}\overline{R} S + P\overline{Q}\overline{R} \overline{S} \\ &\quad + P\overline{Q}\overline{R} S + P\overline{Q}\overline{R} \overline{S} + P\overline{Q}\overline{R} S + P\overline{Q}\overline{R} \overline{S} \end{aligned}$$

$$\begin{aligned} &= 1111, 1110, 1101, 1100, 1011, 1010, 1001, 1000, 1110, 1100, 0110, \\ &\quad 1111, 1011, 0111, 0011, 1101, 0101 \end{aligned}$$

$$= 15, 14, 13, 12, 11, 10, 9, 8, 14, 12, 8, 4, 15, 11, 7, 3, 13, 5$$

$$\Rightarrow \Sigma_m = (3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)$$

$$\begin{aligned} \Rightarrow ABC &= A(\overline{B} + C) \\ &= A(B \cdot \overline{B})(C \cdot \overline{C})(C \cdot \overline{A} + \overline{B}C) \\ &= (A + B \cdot \overline{B} + C \cdot \overline{C})(C \cdot \overline{A} + \overline{B}C) \\ &= (A + B + C)(A + B + \overline{C})(C + \overline{B} + C)(C + \overline{B} + \overline{C})(C + \overline{B} + C)(\overline{A} + \overline{B}) \\ &= 000, 001, 010, 011, 010, 110 \\ &= 0, 1, 2, 3, 2, 6 \end{aligned}$$

$$\pi_m = (0, 1, 2, 3, 6)$$

$$\begin{aligned} \Rightarrow F(A, B, C, D) &= B(B + \overline{D})(\overline{A} + C)(A + C + \overline{D}) \\ &= (A \cdot \overline{A} + B + C \cdot \overline{C} + D \cdot \overline{D})(A \cdot \overline{A} + B + C \cdot \overline{C} + \overline{D})(\overline{A} + B \cdot \overline{B} + C) \\ &\quad (A + B \cdot \overline{B} + C + \overline{D}) \end{aligned}$$

$$\begin{aligned} &= (A + B + C + D)(A + B + C + \overline{D})(A + B + \overline{C} + D)(A + B + \overline{C} + \overline{D})(\overline{A} + B + C + D) \\ &\quad (\overline{A} + B + C + \overline{D})(\overline{A} + B + \overline{C} + D)(\overline{A} + B + \overline{C} + \overline{D})(A + B + C + D)(A + B + \overline{C} + D) \\ &\quad (\overline{A} + B + C + D)(\overline{A} + B + \overline{C} + \overline{D})(\overline{A} + B + C + D)(\overline{A} + B + C + \overline{D})(\overline{A} + \overline{B} + C + D) \\ &\quad (\overline{A} + \overline{B} + C + \overline{D})(A + B + C + D)(A + \overline{B} + C + \overline{D}) \end{aligned}$$

$$\begin{aligned} &= 0000, 0001, 0010, 0011, 1000, 1001, 0010, 0011, 0001, 0011, 1001, \\ &\quad 1011, 1000, 1001, 1100, 1101, 0001, 0101 \end{aligned}$$

$$\Rightarrow 0, 1, 2, 3, 8, 9, 2, 3, 1, 3, 9, 10, 8, 9, 12, 13, 1, 5$$

$$\Rightarrow \pi_m = (0, 1, 2, 3, 4, 5, 8, 9, 10, 12, 13)$$

$$6) f(x, y, z) = (x+y)(y+z)(x+z)$$

$$(x+y+z \cdot \bar{z}) (x \cdot \bar{x} + y+z) (x + y \cdot \bar{y} + \bar{z})$$

$$= (x+y+z)(x+y+\bar{z})(x+y+z)(x+\bar{y}+z)(x+y+\bar{z})(x+\bar{y}+\bar{z})$$

$$= 010, 011, 010, 110, 101, 111$$

$$= 2, 3, 2, 6, 5, 7$$

$$T_m = (2, 3, 5, 6, 7)$$

$$7) F(P, Q, R, S) = \bar{Q} + PR + \bar{R}S + \bar{P}RS$$

$$= (P+\bar{P})\bar{Q}(R+\bar{R})(S+\bar{S}) + P(Q+\bar{Q})R(S+\bar{S}) + (P+\bar{P})(Q+\bar{Q})\bar{R}S + \bar{P}(Q+\bar{Q})RS$$

$$= P\bar{Q}RS + P\bar{Q}R\bar{S} + P\bar{Q}\bar{R}S + P\bar{Q}\bar{R}\bar{S} + \bar{P}Q\bar{R}S + \bar{P}Q\bar{R}\bar{S} + \bar{P}Q\bar{R}S + \bar{P}Q\bar{R}\bar{S} \\ + PQR\bar{S} + PQR\bar{S} + PQR\bar{S} + PQR\bar{S} + PQR\bar{S} + PQR\bar{S} + PQR\bar{S} + PQR\bar{S} \\ + \bar{P}Q\bar{R}S + \bar{P}Q\bar{R}\bar{S}$$

$$= 1011, 1010, 1001, 1000, 0011, 0010, 0001, 0000, 1111, \\ 1110, 1011, 1010, 1101, 1001, 0101, 0001, 0111, 0011$$

$$= 11, 10, 9, 8, 3, 2, 1, 0, 15, 14, 11, 10, 13, 9, 5, 1, 7, 3$$

$$7) \sum_m = (0, 1, 2, 3, 5, 7, 8, 9, 10, 11, 13, 14, 15)$$

K-map (Karnaugh map) :-

- The k map is a chart or a graph, composed of an arrangement of adjacent cells. representation of particular combination of variables in sum of product form.
- Like a truth table k map shows the relation between input's.
- Since k map is a graphical representation of boolean expression.
- At n variable k map will have

(2-variable) $2^2 = 4$ cells, similarly,

(3-variable) $2^3 = 8$ cells,

(4-variable) $2^4 = 16$ cells,